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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Young Children's Behavior in Division Problems submitted by Roger Daniel Bourgeois in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

Appropriate problem-solving situations for young children should be devised if these are to be included in a model for mathematics learning. Knowledge of how children behave when faced with such situations may help to generate suitable guidelines for the construction of problems.

The present study examined the behavior of children, aged three to nine years, as they attempted to solve two mathematically equivalent division problems. These were a cargo groups problem and an animal groups problem. Each problem consisted of a measurement and of a partitive division task.

The subjects were interviewed twice. In the first interviews 60 subjects were given the cargo groups problem and 30 of these were also given the animal groups problem. The second interviews were conducted one year later. In this aspect of the investigation, 44 of the original 60 subjects were given the animal groups problem and 21 of these also attempted the cargo groups problem. The behaviors of the subjects as they solved the problems were recorded on half inch videotape.

The apparatus used with each problem permitted different levels of solution. The subjects exhibited a wide range of solution procedures in attempting the tasks. Some subjects appeared to be working randomly, others used a systematic procedure. Some made groups on the table without associating the objects with those of another set, and some gave the solution without manipulating the material. The procedures changed and involved less manipulation as

the age of the children increased.

Comparison of solution results and procedures employed seemed to indicate that the partitive division problem with the animal groups was more difficult than the measurement division problem with the same apparatus. The structure of the cargo groups problem did not permit a valid comparison between the two processes.

When the same apparatus was used for two different mathematical problems, it was found that the subject's choice of procedure in attempting the second task was influenced at times by the requirements of the first task. The data also revealed that the different structures of mathematically equivalent problems make some easier to solve than others.

The various tasks contained distractions to which many of the subjects responded. The behaviors associated with the distractions included classifying the objects according to certain attributes such as color, kind or size. Other responses to the distractions consisted of making detailed simulations of real world situations.

The children displayed a variety of verbalization behaviors while attempting the problems. These spontaneous verbalizations appeared to be elicited mostly by the material for the younger subjects and seemed to be more task related for the older subjects.

It was suggested by the study that further research be undertaken in certain areas of problem solving, the most important of which appears to be the role of noise in such learning situations.

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Chapter I

THE PROBLEM AND THE NATURE OF THE INVESTIGATION

INTRODUCTION AND STATEMENT OF THE PROBLEM

Problem solving is advocated as an important activity in concept development in many new pedagogical books and teacher guides for mathematics instruction. Although problem solving has been part of the mathematics curriculum for many years, there has been a shift in the last few years from a deductive to an inductive approach. Getzels (1964) referring to problem solving, insight, discovery, inquiry, originality and creativity (which are common expressions in most courses of study today), pointed out that materials and methods advocated are often based more on exhortation and testimonial than on empirical research.

Much research on problem solving in mathematics has been reported (Gorman, 1967; Kilpatrick, 1969; Riedesel, 1969; Suydam, 1967, 1974) but most of the studies were concerned with children's verbal responses to problems. The method usually employed was to present the child with a verbally stated problem where the response was in the form of expressions or statements employing mathematical symbols. In some of these studies, the solutions were analysed for correctness when different modes of presentation were used (Ammon, 1973; Anderson, 1970; LeBlanc, 1968; Steffe, 1970; Steffe and Johnson, 1971). In other studies the strategies employed by the subjects were analysed (Affolter, 1970; Kilpatrick, 1968; Paige and Simon, 1966;

Stern, 1967; Stern and Keislar, 1967), and in others the effect of different problem-solving instructions were compared (Jerman, 1971; Scandura, 1964; Wilson, 1967; Wittrock, 1967).

Little (1974) studied nonverbal behaviors in young boys while solving division problems. He has suggested that further research be undertaken using a larger sample that would include both sexes. He further suggested that a team of investigators be involved with the study and that modifications be made in the method of analysis.

Kilpatrick (1969) and Riedesel (1969) have pointed out shortcomings in many of the studies on problem solving. Kilpatrick indicates that this area of mathematics learning has not been investigated systematically and that investigators do not build on previous research. He proposes that "clinical studies be undertaken as precursors to larger efforts (Kilpatrick, 1969, p. 532)."

If problem solving is to be included in a model for mathematics learning, information regarding how children behave when faced with such situations must first be obtained. This could serve as a guide in devising suitable encounters from which concept formation or abstraction may take place. Such knowledge is lacking at the present time. As Bruner (1973) points out:

> There is a surprising lack of research on how one most wisely devises adequate learning episodes for children at different ages and in different subject matters (p. 422).

If the relationship between problem solving and mathematics learning is to be studied systematically appropriate problem-solving situations must be devised. The construction of these situations should not be based entirely on theories and hypotheses but rather on sound

empirical evidence that certain situations are likely to elicit certain responses or behaviors from young children.

Although criteria for the construction of problems can be formulated, there is no information available at present to indicate how children will react to different aspects of the problem at different age levels. If an element of reality or some other distraction is introduced into a problem, for example, how does a child behave when confronted with the situation? Does he see the mathematical aspect of the problem or is he distracted by the perpetual representation? To what degree does the apparatus or the configuration influence his response?

It is in the search for answers to such questions that the present study was undertaken. If problem solving is to be studied in order to be included in a model for mathematics learning, information must be obtained on how children behave at different age levels when attempting to solve problems. With such data criteria guiding the construction of "good" problems can be formulated.

PURPOSE OF THE STUDY

This investigation is an attempt to gain information on young children's behavior while attempting to solve nonverbal mathematical problems. The main purpose of this study is to observe, record and describe the behaviors of the subjects, ages three to nine, while engaged in division problem-solving situations. The behaviors analysed consist of manipulative actions and verbalizations. The problem-solving procedures employed by the subjects are catalogued and

differences in procedures are compared between age groups for the various tasks. Procedures are also analysed to see which led to a correct solution and which did not. The study is comprised of a cross-sectional and a longitudinal sampling.

BACKGROUND

Since the early sixties psychologists and educators have focused their attention on young children's cognitive development. The child's environment is recognized as an important factor in learning. A proliferation of manipulative materials has become available to the early childhood educator as new emphasis is placed on the role of concrete objects as a stimulus and as an aid to learning.

The theoretical base for many of the changes that have taken place in early childhood education in the last fifteen years can be traced to the writings of Piaget (1950, 1957, 1964), Hunt (1961, 1969) and Bloom (1964). Much of the interest in the learning environment and in didactic material can be attributed to them. Piaget's theory of cognitive development is based on the child's adaptation to his environment. Experiences with physical objects is considered by Piaget (1964) as an important factor in development. He wrote:

> Experience of objects, of physical reality, is obviously a basic factor in the development of cognitive structure (p. 1).

Although Piaget's theory of cognitive development has many implications for the classroom, it does not prescribe how to devise meaningful learning experiences for children. This remains the role of educational research.

Bruner (1966, 1973), Sears (1966), Sava (1973) and White (1975)

have indicated the need for research on how to devise adequate learning episodes for young children. As Bruner (1966) points out:

> It would be more efficient and more useful if embryonic instructional materials could be tried out under experimental conditions so that revision and correction could be based upon immediate knowledge of results (p. 54).

The recourse to manipulative materials in early childhood education at present is based largely on theories and beliefs. Very little is known on the kind of interaction that takes place when children manipulate these materials. Research in this area can be undertaken working from different conceptual frameworks. Working from a theoretical base of behavior reinforcement as derived from Skinner and Gagné, studies could be conducted to investigate the effects of highly structured material, such as the Montessori or the Stern materials, on learning mathematics. An alternative is a theoretical framework of cognitive development as derived from Piaget, Bruner and Dienes.

Working from the latter standpoint Nelson and Sawada (1974), supported by a Canada Council research grant, undertook a longitudinal study into young children's behavior in nonverbal problem-solving situations. The domain of their study was to:

(a) Identify, select, and refine criteria for the specification and construction of mathematical problems which could be used effectively to study the problem-solving behavior of children.

(b) Construct, on the basis of the refined criteria, a selection of problem situations.

(c) Engage children, under carefully controlled conditions,

in interacting with the problem situations.

(d) Observe, record, and categorize the behaviors manifested by the children while interacting with the problem situations.

(e) Trace the scope and sequence of problem-solving behaviors over the age range of three to eight.

The purpose of the present study is to analyse the problem-solving behavior of the subjects while engaged in two of the twelve problem situations devised by Nelson and Sawada.

GUIDELINES

A first step in the investigation was to establish guidelines for the designing of problem situations that would be suitable to explore children's behavior. Nelson and Sawada (1974) adopted as guidelines the characteristics of a "good" problem as outlined by Nelson and Kirkpatrick (1975). These characteristics are:

1. The problem should be of significance mathematically.
2. The situations in which the problem occurs should involve real objects or obvious simulations of real objects.
3. The problem situation should capture the interest of the child.
4. The problem should require the child himself to move, transform, or modify the materials.
5. The problem should offer opportunities for different levels of solution.
6. The problem situation should have many physical

embodiments.

7. The child should be convinced that he can solve the problem, and he should know when he has a solution for it.

The works of Piaget, Bruner, Dienes and others provided initial guidance in designing these guidelines.

The Problem Should Have Mathematical Significance

If the child is to learn mathematics he should be exposed to mathematical relationships and concepts. Bruner (1971) states:

> . . . the best introduction to a subject is the subject itself. At the very first breath, the young learner should, we think, be given the chance to solve problems, to conjecture, to quarrel as these are done at the heart of the discipline (p. 60).

Dienes (1960) regards mathematics as a structure of relationships and the learning of mathematics as the apprehension of the relationships between the concepts, being able to communicate with the symbolism and the ability to apply the resulting concepts to real situations in the world.

Dienes (1960, 1973) distinguishes several instructional stages in concept formation. He advocates a play stage as a first step in the learning process. The ingredients of a concept are played with during this stage in what he calls preliminary games. This serves as a period of adaptation during which the child comes to realize certain constraints in a situation. The child, according to Dienes, is then ready for the second stage where he is able to play with the restrictions which have been imposed, that is to say the rules of the game. The set of rules will depend upon the relevant mathematical ideas that we wish the child to learn.

Dienes' structured discovery approach to mathematics learning is an object-game-oriented approach. He contends that:

> it is necessary to create a suitable environment through which the child will ultimately learn the mathematical ideas in question (Dienes, 1973, p. 10).

Dienes principles of mathematics learning can easily be translated into a problem-solving approach using structured situations without using highly structured material.

Nelson (1974) states:

> In my view the central purpose of any mathematical instruction at the early childhood level is to help the child see order and meaning in situations and events which occur in his everyday activities (p. 11).

It seems reasonable to assume that useful problem-solving situations can be devised which will get children involved in the process of mathematics. These situations can be made to embody mathematical concepts without involving complicated mathematical symbols.

The Situation Should Involve Real Objects

If the problem situation is to be comprehensible and have meaning for the child it should involve concrete objects and it should relate to his world of reality. Without real objects the child would have to solve the problem mentally and give verbal responses which he may be incapable of doing. Sinclair (1971) points out how the six- or seven year-old child often distorts reality, and if this is so, she asks how he apprehends information which the adult presents to him in a representational manner (p. 126). According to Piaget, the child "comes to see the world as coherent, as structured, to the extent that he acts upon the

world, transforms it, and succeeds in coordinating these actions and transformations (Duckworth, 1964, p. 2)." Hunt (1969) adds:

> . . . spoken language can have little symbolic value until after images, or the central processes representing objects, persons, events, and relationships, have been developed through concrete perceptual encounters with those objects, persons, events, and relationships (pp. 42-43).

Dienes also recognizes the importance of the child's contact with the environment in mathematics learning. He states:

> . . . mathematics is based on experience; it is the crystallization of relationships into a beautiful regular structure, distilled from our actual contacts with the real world (Dienes, 1960, p. 11).

Contact with the world can only be assured if the child has real objects at his disposal. But as the natural environment may not always provide for the required attributes, the problem situation will often have to consist of simulations of real objects and of real situations, or as Dienes (1960, 1973) puts it an "artificial" environment.

The Problem Situation Should Capture the Interest of the Child

Motivation in children's learning is a prime concern of educators. Bruner (1960) argues for intrinsic motivation based upon the arousal of interest in what is to be learned rather than on some external goal. Hunt (1969) hypothesizes that the child will be interested in a situation if the circumstance he encounters is relevant to the information that he has stored and to the skills that he has already developed. According to Hunt (1961) success and pleasure depend upon the gap that exists between the child's expectation and abilities and the task required. If the gap is too large

it evokes distress and withdrawal; if no gap exists, the child is not motivated and may be bored resulting in no development. On the other hand, if the discrepancy is within the limits of the child's capacity for accommodation it will be a source of interest or curiosity.

Bruner (1964) points out that little is known at present on "the level of uncertainty and tension that must be present to initiate problem-solving behavior and what is necessary to keep active problem solving going (p. 314)." Hunt (1969) prescribes preschool enrichments that would provide children with "a wide variety of objects and of models of actions and relationships among objects and actions (p. 43)."

In devising problem-solving situations for young children one must hypothesize as to what characteristics these should have in order to capture the interest of the child so that he may interact and attempt to solve the problem. Appropriateness of a particular problem situation for a certain age level may then be determined through research. Nelson and Kirkpatrick (1975) state that interest in a problem can be achieved "by the nature of the materials, by the situation itself, by the transformations the child can impose on the materials, or by some combination of these factors (p. 72)."

The Child Should Move, Transform, or Modify the Materials

The importance of active participation on the part of the child in the learning process has been emphasized by Piaget (1964, 1970a, 1970b, 1973a, 1973b). He points out that not only should the child have concrete materials at his disposal but that he should

perform actions upon them. He states:

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it (Piaget, 1964, p. 8).

Piaget (1964, 1970a) distinguishes between physical experience and logical-mathematical experiences. The former consists of acting upon an object and deriving knowledge from the object itself. Piaget (1970a) refers to abstraction derived from such an experience as simple abstraction. In logical-mathematical experience knowledge is not drawn from the object that is acted upon, but from the action carried out upon it. Piaget considers this to be the basis of logical and mathematical abstraction. This kind of abstraction is referred to as reflective abstraction (Piaget, 1970a, pp. 16-17).

If problem solving is to serve as a vehicle for mathematical concepts, it is important that materials be made available and that the child be encouraged to manipulate them. Abstractions may take place through the transformations that the child brings to them. Piaget wrote:

> It is absolutely necessary that learners have at their disposal concrete material experiences (and not merely pictures), and that they form their own hypotheses and verify them (or not verify them) themselves through their own active manipulations (Piaget, 1973b, p. x).

The Problem Should Offer Opportunities for Different Levels of Solution

Piaget has shown how a child passes through different stages of development and how his comprehension of the world changes as he

adapts to his environment. Adaptation for Piaget (1953), is accomplished when equilibrium is established between the co-occurent processes of assimilation and accommodation. According to Piaget (1950), the child assimilates information from the environment and accommodates his cognitive structure to the new situation so that equilibrium is restored.

Hunt (1961) agrees with Piaget that it is important to base educational practice on the natural phases of the child's interaction with the environment. He considers however that "environmental circumstances force accommodative modifications in schemata only when there is an appropriate match between the circumstances that a child encounters and the schemata that he has already assimilated into his repertoire (p. 268)." According to Hunt, the notion of the proper match between circumstance and schema is crucial for education.

Rather than attempt to fit the circumstance to match the child's schemata at a particular stage of development as Hunt (1961) advocates, it appears more appropriate to provide circumstances that can be approached from different aspects and provide different levels of interaction. This would permit the child himself to select the appropriate interaction.

Bruner (1960) points out that the child has a characteristic way of viewing the world and of explaining it to himself at different stages of development. He identifies three parallel systems for processing and representing information. One is through manipulation and action (enactive), one is through perceptual organization and imagery (iconic), and the other is symbolically (Bruner, 1966, p. 28).

Mathematical problems devised for young children should permit solution at any of these levels so that the child may bring his experience to bear on the situation and thus choose his own level of interaction.

The Problem Situation Should Have Many Physical Embodiments

This criterion would appear to apply to a series of problems rather than to an individual problem. The importance for the learner to encounter a mathematical concept in various, apparently different, situations has been acknowledged by several authors (Bruner, 1973; Dienes, 1960, 1964a, 1973; Skemp, 1971; Van Engen, 1953).

Dienes considers this aspect of learning mathematics of prime importance. One of the requirements for insightful mathematics learning according to Dienes (1964a) is that the child encounters many varied situations out of which the common mathematical properties can be extracted. He believes that the "situations should be as different from each other as possible, in order that the irrelevancies should be fully dispensed with (p. 183)." This implies that the child be provided with "tasks which look quite different but have essentially the same conceptual structures (Dienes, 1960, p. 43)."

The nature of concept formation, in Dienes' thinking, is based on abstraction and generalization. He considers much of mathematical thinking as a blend of the two processes. Abstraction is regarded as a constructive operation and can take place by abstracting information from many different concrete situations. Once an abstraction has taken place, Dienes contends that it allows us to apply the abstracted structure to many different types or kinds of situations. The extension of an abstracted concept to different situations is generalization.

In contrast with Piaget, Dienes does not differentiate between simple and reflective abstraction. He states:

> Abstraction takes place as a result of a selective process. When we abstract a property of events, we consider all events with the property as belonging together in some way and all other properties of such events as being for the moment irrelevant (Dienes, 1964a, p. 181).

Dienes contends that abstraction may take place if a concept is encountered in various contexts. It appears, however, that abstraction would also depend upon the kind of interaction that takes place in the problem situation. If reflective abstraction is to occur, the situation should provide for the child to draw the knowledge not from the objects but from the actions imposed upon them.

The Child Should Know that He can Solve the Problem and He Should Know When He Has a Solution

If the child is to interact meaningfully with the problem situation he must first of all be convinced that he is able to solve the problem. Bruner (1971) puts forth the conjecture that most young children in school spend much time and effort in attempting to figure out what it is that is required of them. Polya (1957) identifies the first step in problem solving as the ability to see clearly what is required.

Problems may be best understood by young children if they are presented in the form of a task to be accomplished with certain manipulative materials. Once the child has transposed or transformed the material he can know when the task is accomplished and thereby have a solution to the problem. His solution may not be correct but he will be convinced that he has accomplished the task and has found a solution. It is important for the child to know when he has a solution. Bruner (1966) states:

> To achieve the sense of accomplishment requires a task that has some beginning and some terminus (p. 119).

ASSUMPTIONS

1. It is assumed that the problem situations permit a wide range of solution procedures.
2. It is assumed that the behavior of the child in a clinical situation is indicative of the way he would tackle a problem in his daily experiences.
3. It is assumed that the problems and the materials dictate the range of behavior.
4. It is assumed that the behaviors which have been recorded can be analysed in a meaningful and systematic way.
5. It is assumed that from the recorded data it will be possible to trace the nature and development of behaviors of children in problems requiring division.

LIMITATIONS OF THE STUDY

1. The behaviors will not constitute a complete profile of mathematical behavior at any age level. The material and the problem situations limit the range of mathematical processes.
2. The total background of each subject will not be taken into consideration. No attempt was made to obtain background information on the subjects.
3. The behaviors of the subjects will be characteristic of this sample only.
4. When the data were taped the subjects were confined in a relatively small space.

CLARIFICATION OF TERMS

Measurement division. A measurement division problem is one in which the number of subsets is to be determined when a given set of elements is separated into equivalent subsets.

Partitive division. A partitive division problem is one in which the number of elements in each subject is to be determined when a set of elements is separated into a given number of equivalent subsets.

Problem. A verbal statement about the apparatus requiring a response from the subject.

Problem situation. All aspects of the apparatus designed for the problem.

Problem-solving behavior. The overt action manifested by a child's interaction with the material and his verbal expressions.

SIGNIFICANCE OF THE STUDY

Although much research on problem solving has been done in the past very few of the studies undertaken with young children took a non-verbal approach. It would seem reasonable to assume however that non-verbal responses would be more indicative of the way children perceive a task. The present study should furnish valuable information on the kind of processes available to young children when attempting to solve problems.

The present study is deemed important for four reasons.

1. The categorization and analysis of behaviors should lead to the identification of strategies that are available

to young children in problem solving.

2. The findings should provide information on the development of certain problem-solving procedures in children.

3. The findings should serve as a guide in devising concrete situations in research intended to study the role of problem solving in mathematics learning.

4. The analysis of the behaviors exhibited by the subjects in the two division problem situations will be coordinated with the analysis of the behaviors exhibited in the other problems being studied in the project.

OUTLINE OF THE STUDY

Chapter II contains a detailed description of the two division problems investigated and the mathematical basis of each problem. The sampling, data collection and analysis procedures are also explained.

The findings of the study are reported in Chapter III.

Chapter IV provides a summary and a discussion of the findings, implications of the investigation, and suggestions for further research.

Chapter II

DESIGN OF THE INVESTIGATION

Twelve problem situations were designed by Nelson and Sawada (1974) for their investigation. Six were based on number relationships and six on spatial relationships. Six of the twelve problems were presented individually to each child who had at his disposal apparatus specific to each. The behaviors of the subjects while attempting to solve the problems were recorded on videotape.

PURPOSE OF THE STUDY

The basic purpose of this study is to investigate problem-solving behaviors of young children.

The specific purposes are:

1. To encode the behaviors exhibited by the subjects while they were engaged in solving
   (i) the cargo groups problem,
   (ii) the animal groups problem.
2. To catalogue and analyse these behaviors.

THE PROBLEMS

The Cargo Groups Problem

The apparatus consisted of a masonite base 76 cm x 106 cm x 6 mm. At each end of the board 4 mm masonite sections, approximately 76 cm x 40 cm, were glued leaving a winding strip (the river), approximately 35 cm wide, across the middle of the board.

Two kidney shaped islands in the middle of the river were made from 3 mm masonite glued to the baseboard. The river area was painted light blue and the islands were green. On each side of the river a large parking lot was painted black as was a roadway leading away from each parking lot. The remainder of the board was painted green.

There was a ferry boat, of 11 cm x 6.5 cm x 3 mm masonite with balsa wood sides, with one end convex and the other concave. The apparatus also consisted of 15 plastic toy cars of various colors,

5.3 cm x 2 cm. Three wooden blocks, one blue, one red and one brown, 5 cm x 5 cm x 2.5 cm, were used to represent houses.

The cars were placed in the parking lot closest to the child and he was asked how many trips would be required for the ferry boat to take all the cars across to the other parking lot if the ferry took three cars at a time. After the problem had been solved to the child's satisfaction, he was asked how many cars would be at each house if there were the same number at each house. (See Appendix A for detailed protocols.)

<u>The Animal Groups Problem</u>

The apparatus consisted of (i) a base board, (ii) posts, (iii) walls and (iv) animals.

The baseboard, of 2 cm plywood, had dimensions 76 cm x 107 cm.

Twenty-five holes formed a square array with centres 18 cm apart and were situated symmetrically on the board. The board was painted dark green.

Each post had four slots placed symmetrically around the circumference and the diameter was reduced at one end to fit the holes of the baseboard. The walls were made of 16.5 cm x 7.5 cm x 6 mm masonite slabs which could easily slide into the post slots. Twenty plastic and rubber toy animals (3.8 cm to 5 cm tall) completed the apparatus.

The set of 20 toy animals was presented to the child and he was asked to build enough cages so that there would be five animals in each. After the task was completed, some cages were dismantled, or some were built, so that there were three and two animals were removed. The child was now asked how many animals would be in each cage if there were the same number in each. (See Appendix A for detailed protocols.)

THE MATHEMATICS OF THE PROBLEMS

The two problem situations were designed in such a way as to embody equivalent mathematical concepts. Each problem consisted of a measurement division task and of a partitive division task.

Measurement Division

A measurement division problem is one in which a given set of objects is to be separated into smaller subsets of a given size and the total number of equivalent subsets is to be determined (Zlot, 1976, p. 132). In the cargo groups problem, it was required to find

how many groups of three cars could be formed with the set of 15. The mathematical statement for this problem is

$$15 \div 3 = n.$$

The animal groups task consisted of determining how many cages were required in order to place the 20 animals into cages in such a way as to have five in each cage. The mathematical statement can be written

$$20 \div 5 = n.$$

The solution to these problems can be obtained mathematically

(i) by repeated subtraction

(a) <u>cargo group</u>

	number of groups
15	
- 3	1
12	
- 3	1
9	
- 3	1
6	
- 3	1
3	
- 3	1
0	5

(b) <u>animal group</u>

	number of groups
20	
- 5	1
15	
- 5	1
10	
- 5	1
5	
- 5	1
0	4

(ii) by determining

(a) what factor multiplied by 3 gives a product of 15

$$5 \times 3 = 15$$

(b) what factor multiplied by 5 gives a product of 20

$$4 \times 5 = 20$$

(iii) by recall

(a) <u>cargo group</u>

$$\begin{array}{r} 5 \\ 3\,\overline{)\,15} \end{array}$$

(b) <u>animal group</u>

$$\begin{array}{r} 4 \\ 5\,\overline{)\,20} \end{array}$$

The same mathematical relationship, or concept, was involved in the two perceptually different situations. This aspect of the problems is often referred to as the principle of multiple enbodiment (Dienes, 1964b, p. 40). The variables involved in both measurement division problems, however, were different. Dienes refers to this aspect of the problems as the principle of contrast or the mathematical variability principle (Dienes, 1964b, pp. 40-41).

Partitive Division

In partitive division the elements of a given set are to be separated to form a given number of equivalent subsets, and the number of objects in each subset is to be determined (Zlot, 1965, p. 124). In the cargo groups problem it was required to find how many cars would be at each of three houses if the cars were partitioned equally among them. The mathematical statement can be written

$$15 \div n = 3.$$

In the animal groups problem it was required to determine how many animals would be in each of three groups if 18 animals were partitioned equally. The mathematical statement is

$$18 \div n = 3.$$

The mathematical solution to these problems can be obtained in the same manner as for the measurement division problems. The multiple embodiment principle and the mathematical variability principle also apply for these two problems.

The variables in the measurement and partitive division problems for the cargo groups tasks remain the same while they vary in the two animal groups tasks. It is therefore possible to see a

mathematical relationship between the measurement and partitive division problems for the cargo groups tasks, whereas it is not the case for the animal groups tasks.

SOLUTION LEVELS

The problems were presented in the form of a task which was a simulation of a situation that could be encountered in the real world. The problems were designed in such a way as to permit different levels of solution.

As has been pointed out earlier, Bruner (1966) identifies three systems for processing information and for representing it. The measurement division tasks in this study permitted solution to be obtained by any of these processes.

(i) Enactive

(a) A child could arrive at a solution by physically placing three cars on the ferry at a time and making five crossings, or a child could build four cages and place five animals in each.

(b) A child could make five groups of three cars or four groups of five animals on the table and then count the number of groups.

(ii) Iconic

The solution could be arrived at by visualizing five groups of three cars or four groups of five animals.

(iii) Symbolic

A solution could be obtained by counting the number of objects in the given groups and then divide mentally

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

by three or by five.

The partitive division tasks permitted solution at the enactive and symbolic levels. Solution at the iconic level is not considered possible for the children on account of the large number of objects involved. A child could not visualize the groups because it was not known how many objects were to be in each, indeed that was the statement of the problem.

(i) Enactive

(a) A child could systematically distribute the cars one at a time at each house or the animals in the cages and then count the number of objects in one group.

(b) A child could distribute the objects unsystematically and arrive at the answer by trial and error.

(c) A child could make groups on the table and determine how many objects would be in each group.

(ii) Symbolic

In order to solve the problem on this level a child would have to count the number of objects in the given group and then divide by three.

RESEARCH PROCEDURES

The Sample

Six hundred letters were distributed to parents of children in two day-care centres and in grades one, two and three in five Edmonton schools in the vicinity of the University. The project was outlined in the letter and the parents were invited to consent to

their child's participation during the summers of 1974 and 1975. An application form was enclosed with the letter. Of the approximately 200 affirmative replies, a sample of 15 subjects at each age level from three to eight was randomly chosen. The detailed account of the method of selection has been given by Little (1976).

Fifteen sets, each consisting of six problems were systematically established for the cross-sectional sampling. (See Appendix D.) Each set was comprised of four different problem situations and two problems equivalent to two of these. Each problem set was randomly assigned to one subject in each age group. This provided for each of the six different problem situations to be presented to ten subjects and each of the six equivalent problems to be presented to five subjects in each age group. A schedule of appointments was established and the interviews started on June 24 and terminated on July 23, 1974.

The theoretical distribution of the problems for the longitudinal sampling provided for subjects to be presented with the equivalent to each problem that they had attempted the previous year. Subjects interviewed in 1975 were to be assigned the same problem set number that they had been assigned for the cross-sectional sampling. (See Appendix D.) This distribution provided for each of the six equivalent problems to be presented to ten subjects and for each of the six different problems to be presented to five subjects in each age group. The distribution of the problems also provided for each different problem situation and its equivalent to be encountered by five subjects in each age group in both the cross-

sectional and the longitudinal sampling.

The parents of those children who had participated in the cross-sectional sampling were contacted in May and June 1975 and it was possible to obtain 74 of the original subjects to participate in the longitudinal sampling. Because the number of subjects was reduced from 90 to 74 for the longitudinal sampling, only 21 subjects attempted the cargo groups problem and 44 attempted the animal groups problem. (See Appendix D.) The interviews for this phase of the study were conducted from July 3 to July 25, 1975.

The Pilot Study

A pilot study involving a number of children from three to eight years of age was carried out prior to the data collection. Little (1976) has outlined the purposes of this pilot study.

A second pilot study was carried out in June 1975 in order that new support staff members could be made familiar with the interview operations. The recording techniques and the protocols were further refined for the longitudinal data collection. Minor changes in the protocols guaranteed greater consistency between a problem and its equivalent. (See Appendix A.)

The Interviews

Five interviews were usually scheduled for each recording day, three in the morning sessions and two in the afternoon. Most younger subjects were scheduled for the mornings in order to avoid the effects of fatigue.

Seven adults were normally in the room during the interview:

two cameramen, the interviewer, two assistants, the director or the associate director of the project and a parent. The subjects did not appear to be distracted by the recording instruments nor by the presence of the adults once they became involved with the problems.

Each problem situation was presented to each child individually. The material was placed on a low table in front of the child so that he had easy access to it. The interviewer sat at the child's right and adhered to the protocol. The protocol for each problem provided for an introductory period during which the child could familiarize himself with the apparatus. Once a problem was completed to the child's satisfaction, the apparatus was removed by one of the assistants and the material for the next problem was introduced. A taping session took from one half hour to one hour. Younger children tended to take longer than the older ones to accomplish the various tasks.

Two persons conducted the interviews in the cross-sectional data collection while one interviewer was used in the longitudinal portion of the project.

The Recording

A Javelin videotape recorder and two Sony video cameras were used to record the behaviors of the subjects on half inch (new format) magnetic tape. The cameras and the table where the subjects were seated formed the three vertices of an imaginary triangle; the cameras were approximately three metres from each other and about two metres from the table. Either camera could be used to capture the behavior of the subjects and the split screen technique was at times

employed in order to obtain simultaneous recordings of an action from different angles. At other times this technique was used to record the actions of the subject and his facial expression. A microphone was suspended from the ceiling in order that the subject and the interviewer's verbalizations could be recorded.

All the data from the 90 children who participated in the cross-sectional study are stored on 54 one hour videotapes and the data from the longitudinal study are stored on 45 one hour tapes.

Coding Procedures

Analysis of children's behaviors generated by the two problem-solving situations required a system that would permit a cataloguing of those behaviors. After carefully viewing several tapes, many reoccurring behaviors deemed relevant to the study were identified. A coding system that would enable a transcript of the behaviors was devised. (See Appendix B.) This system was used to encode the behaviors that had been recorded on videotape.

The symbols employed in the code provided for any action or sequence of actions on the part of the subjects and the interviewer to be recorded. In order to avoid losing important information all verbalizations, except for those provided for in the protocol were recorded verbatim. Some of the verbalizations, especially those of younger subjects, were in part inaudible or at times indistinct because of other sounds.

The encoding of the behaviors required the tapes to be played and replayed several times. The time required to accomplish this for each problem ranged from one half hour for some of the older subjects

to two hours for some of the younger subjects. Sample sheets of the recordings may be found in Appendix C.

During the cataloguing of the behaviors several tapes were viewed again to assure correct interpretation of the encoded data. A further check on the reliability of the experimenter's observations is provided by a comparison of his observations with those of two other observers. Two graduate students were enlisted to view tapes and their observations are correlated with those of the experimenter.

The procedure employed was to select one subject randomly from each age group and have the two observers encode the behaviors of the subjects independently. Before commencing, the coding procedure was explained and each observer encoded the behaviors of one subject. The recording sheets were compared with those of the experimenter and areas of disagreement were identified. The data used for this aspect of the study were the cross-sectional sampling for the animal groups problem.

THE ANALYSIS OF DATA

The behaviors of the subjects as they interacted with the material in the various problem-solving situations were recorded on videotape. In the present study the data from two of the twelve problems are encoded and analysed. These two mathematically equivalent problems are the cargo groups and the animal groups problems. The behaviors exhibited by the subjects consisted of physical manipulations and verbalizations.

Cross-Sectional Data

The actions of the subjects while attempting to solve the problems were encoded and their spontaneous verbalizations were recorded verbatim. Solution procedures are catalogued and comparisons are made between subjects and between tasks. The subjects' responses to distractions inherent in the various tasks are identified and compared across the age range. Verbalizations are classified and comparisons made between age groups and between tasks.

Longitudinal Data

The longitudinal data serve as a check on the validity of the findings in the cross-sectional analysis. Data from the second sampling are analysed in the same manner as those of the cross-sectional sampling. The procedures employed by the subjects are compared with those of the first sample and new ones are identified. The responses to the distractions and the verbalizations are also compared with those of the cross-sectional sample.

Chapter III

RESULTS OF THE INVESTIGATION

The main purpose of the study was to observe, record and describe the behavior of the subjects while they were engaged in four different division tasks. It will be recalled that 60 subjects in the cross-sectional sampling performed the cargo groups problem and 30 of these performed the animal groups problem as well. The distribution for the longitudinal sampling was 21 and 44 respectively. Both problems consisted of a measurement division task and a partitive division task. The findings of the investigation are reported in this chapter.

The chapter is composed of two major parts each with five sections as follows:

Part 1. Cross-sectional sampling.

Section 1. Procedures used by students in solving the measurement division problems of cargo and animal groups.

Section 2. Procedures used by subjects in solving the partitive division problems of cargo and animal groups.

Section 3. Comparison of behaviors in the measurement problems with those in partitive problems.

Section 4. Response of subjects to distracting

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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[illegible]

[illegible]

[illegible]

elements in the problem situations.

Section 5. Verbalizations of subjects as they engaged in the problem situations.

Part II. Longitudinal Sampling.

Section 1 to section 5 as for cross-sectional sampling.

A short section is then devoted to a discussion of the reliability of the observations. The chapter ends with a summary of the findings in point form.

Part I. CROSS-SECTIONAL SAMPLING

Section 1. Measurement Division

Cargo Groups

Ten subjects at each age level were given the cargo groups problem except at age three where nine subjects attempted the tasks. One three-year-old made no attempt to do the problems.

The subjects were presented the set of cars and the measurement division task consisted of finding how many groups of three could be made with this set (see Appendix A for protocol). After observing the different methods used by the subjects in attempting to solve the problem, the procedures were categorized as follows.

I. The subject did not attempt the task or was unable to follow the instructions so abandoned the task.

II. The subject used an invalid procedure and failed to arrive at a solution.

III. The subject placed an incorrect number of cars on the

ferry for one or two trips, was prompted, corrected the number of cars but failed to arrive at a solution.

IV. The subject placed three cars on the ferry, made five trips and failed to give the solution.

V. The subject placed three cars on the ferry, made five trips and gave the solution.

VI. The subject placed three cars on the ferry and gave the solution after making only two or three trips.

VII. The subject grouped the cars in threes and gave the solution without making any crossing at all.

VIII. The subject looked at the apparatus and gave the correct solution without manipulating any of the material associated with the problem.

The procedures are listed in the order that was judged to be in increasing level of organization. The distribution of these procedures according to age and sex is given in Table 1. All subjects who were unable to solve the problem were classified in one of the first four categories (N = 35). Those subjects who successfully solved the problem were classified in categories V to VIII (N = 25).

If the classification of the responses does represent solutions which would require increasingly higher levels of organization on the part of the subjects, then the older subjects tended to use the higher levels. At least the majority of the seven-year-olds and all of the eight-year-olds could solve the problem. Almost all three-, four- and five-year-olds failed to obtain a solution.

Twenty-six subjects, representing all age levels,

[illegible]

[illegible]

[illegible]

Table 1

Distribution of Procedures for Cargo Groups—Measurement Division

Procedure Category	3	4	5	6	7	8	
VIII					M M M	F F F	Solution N = 25
VII				F	M M M F	F F	
VI					M	M	
V	F		F	M M F F		M M F F	
IV	F	M M M F F	M M F F F F	M M	M M		No Solution N = 35
III	M M	M M M F	M F F	M M			
II	F F F	M		M			
I	M M M						

Age

M - male
F - female

systematically placed three cars on the ferry and made five crossings (categories IV and V). Although these subjects worked systematically, 15 of them were unable to state that the ferry had taken five trips. The majority of four- and five-year-olds who used this systematic procedure were in this category. Most four-year-olds did not attempt to say how many trips it took while the majority of five-year-olds gave an incorrect answer. Two six-year-olds and one seven-year-old also gave a wrong answer. Another seven-year-old subject indicated he made so many crossings that he forgot how many trips it took.

The ferry was large enough to carry more than three cars and many subjects started by placing four cars on it. This behavior was observed even among six-year-olds. Twelve subjects in all had to be prompted before they placed the correct number of cars on the ferry (category III). None of these subjects gave a correct solution to the problem. When a subject placed four cars on the ferry, the interviewer would ask, "How many cars were to be placed on the ferry?" Most subjects would then remove one car, but four subjects, one age three, two age five, and one age six, indicated that the ferry could take more than three cars. The interviewer replied that the ferry would sink if more than three cars were placed on it. All subjects appeared to accept this rule and proceeded to remove one car from the ferry.

The widest range of behaviors occurred at ages three and six. Most three-year-olds seemed unable to follow the instructions. Some subjects in this age range took two, four, or six cars across at a time. Some took cars to an island and parked them there. Other

subjects brought cars back to the first parking lot. Only one subject correctly answered that it took five trips to get all the cars across.

The response procedures at the six-year-old level varied from an inability to follow instructions to making five groups of three and giving the solution without actually moving the cars across. Half of the subjects at this age level were unable to solve the problem correctly. Two six-year-old boys made incorrect predictions before undertaking the task. One had said that it would take six trips and the other had predicted that it would take 20 trips. Both subjects completed the task and maintained that it took 6 and 20 trips respectively. The boy who said that it took six trips had parked the cars in the second parking lot in such a way that the integrity of the groups was maintained. Only one other subject, a five-year-old girl, parked the cars in this manner. When asked how many trips it took, she counted the groups and gave a correct solution.

An important observation in this problem is that children at every age level interacted freely with the apparatus. Only a few subjects at ages seven and eight chose to solve the problem without manipulating the material. Many subjects who used a valid procedure, however, were unable to solve the problem.

<u>Animal Groups</u>

Five of the subjects at each age level who performed the cargo groups problem also performed the animal groups problem. In the first of the two tasks the children were presented with a group of animals and material to build cages (see Appendix A for protocol). The subjects were asked how many cages it would take in order to put

five animals in each cage. The observed response behaviors were categorized as follows.

I. The subject did not attempt the task or was unable to follow instructions and the task was abandoned.

II. The subject built more than four cages and distributed the animals unequally among them.

III. The subject built fewer than four cages and distributed all the animals among them.

IV. The subject built four cages and distributed the animals unequally.

V. The subject built four cages or fewer, placed five animals in each, built more cages if required and arrived at a solution.

VI. The subject built five cages and then proceeded to place five animals in each of four cages.

VII. The subject built two or three cages, placed five animals in each and gave the solution.

VIII. The subject made four groups of five animals without building cages and gave the solution.

Although the upper categories were judged to be higher levels of solution procedures than the lower categories, the rank order may not be absolute. For example, categories III and IV are not necessarily different levels of solution but are quite different procedures.

The distribution of the different procedures according to age and sex is given in Table 2. The subjects who failed to solve the

[illegible]

Table 2

Distribution of Procedures for Animal Groups—Measurement Division

Procedure Category	3	4	5	6	7	8	
VIII					M	F	
VII		F	F			M F	Solution N = 17
VI		F			M M	M F	
V		M	F F	F	M M		
IV	M			M F			
III		M	F	M			No Solution N = 13
II	F	M	M	M			
I	M M F						

Age

M - male
F - female

problem correctly were classified in categories I to IV (N = 13), while those who were successful were placed in categories V to VIII (N = 17).

A striking feature of the table is the fact that the majority of four- and five-year-olds were classified in the upper four categories while most six-year-olds were classified in the lower categories.

The table shows that the majority of the subjects went through the motions of placing all the animals in cages. Only two subjects grouped the animals and gave the solution without building cages. Four other subjects gave the solution after building only a few cages.

The procedure described in category V may appear to represent a higher level of solution than the procedure classified in category VI. However, none of the subjects classified in category V systematically built one cage, placed five animals in it and then continued the procedure until all the animals had been used. These two categories may not necessarily be different levels of solution for the subjects in the sample but may primarily represent different procedures. It should be noted, however, that in order to solve the problem, the subjects who had built five cages had to overcome the distraction of having more cages than were required.

Table 2 seems to indicate that the four- and five-year-olds were fairly successful with the measurement division task for this problem. The table also shows that there were a greater number of successes at each of these age levels than at age six. The range of

procedures at ages four and five varies rather extensively. A four-year-old subject built six cages and distributed all the animals among them. A five-year-old also built six cages, but he placed all the animals in two of the cages. One subject at each of these two age levels was able to give a correct solution after building only two or three cages.

Only one six-year-old successfully solved the problem. The procedures employed by the subjects in this age range also varied greatly. One subject who built six cages finished the task with one animal in two of the cages and the remaining animals distributed among the other four cages. Another subject built four cages and placed 11 animals in the first cage, six in the second, two in the next and one in the last. The only other subject to place an animal alone in a cage was a three-year-old boy who isolated the lion. Many of the subjects who attempted the animal measurement division task appeared to classify the animals. Behaviors involving classification are discussed later in this chapter.

In comparing the results of the two measurement division tasks, it is noted that 25 subjects out of 60 correctly solved the cargo groups problem while 17 out of 30 were able to solve the animal groups problem.

Comparing the number of successes for the two tasks in the different age groups would seem to indicate that the four- and five-year-olds had more success with the animal groups task and that the six-year-olds were more successful with the cargo groups task. No four-year-old and only one five-year-old correctly solved the cargo

groups task. Only half as many subjects attempted the animal groups task and three subjects in each of the four- and five-year-old age groups were successful with this problem. Half of the six-year-old subjects were successful with the cargo groups task but only one out of five correctly solved the animal groups task. All seven-year-old subjects correctly completed the second task while two failed to complete the first task.

Section 2. Partitive Division

After completing the measurement division task with the cargo groups, or with the animal groups, each subject was asked to solve a partitive division problem (see Appendix for protocol).

Cargo Groups

The second task with the cargo groups consisted of partitioning 15 cars among three houses. The cars that had been taken across the river in the first task were left where the children had parked them. The subjects were now asked how many cars would be at each house, if they were parked so that there would be the same number at each house. The observed solution procedures were categorized as follows.

I. The subject did not attempt the task or the task was abandoned.

II. The subject randomly placed the cars on the road.

III. The subject randomly and incorrectly distributed the cars at the three houses.

IV. The subject used some systematic procedure in distributing the cars at the houses but failed to

arrive at a correct solution.

V. The subject distributed the cars randomly or redistributed the cars and arrived at a correct solution.

VI. The subject placed two, three or four cars systematically at each house, then distributed the remainder correctly.

VII. The subject distributed the cars one at a time in order around the three houses.

VIII. The subject placed five cars at a time at each house.

The listed procedures are in the order that was deemed to represent the lowest level to the highest level of solution.

Each subject was classified as belonging to one of the eight categories according to the procedure he or she employed while trying to solve the problem. Table 3 shows the distribution of the different procedures according to age and sex. All the subjects who did not arrive at a correct solution (N = 26) are included in the first four categories. The subjects who solved the problem correctly (N = 34) were classified in the last four categories.

Table 3 indicates that there is a relationship between age and the type of procedure employed. Most of the younger subjects were classified in the lower categories while the older subjects were classified in the upper categories.

The highest frequencies are in categories IV and VI. The procedures in these two categories suggest a systematic approach in the attempt to solve the problem. Category VII also involves a very systematic approach. Some three-year-olds employed some form of

Table 3

Distribution of Procedures for Cargo Groups—Partitive Division

Procedure Category	3	4	5	6	7	8	
VIII					M M F	M F F	
VII			F	M M F	M	M F F F F	Solution N = 34
VI		F	M F	M M F	M M M M	M F	
V		F	M M F		M M		
IV	F F	M M M F	F F F F	M F			
III	M F	M M		M			No Solution N = 26
II	M			M			
I	M M M F F	M M					

Age

M - male
F - female

[illegible]

systematic procedure but they were unable to solve the problem. The number of subjects working systematically increases with age. It is not before age six however that the majority of the subjects using such a method achieved success with the problem. All eight-year-olds appeared to work very systematically.

A distinction was made between those subjects who distributed the cars at the houses several at a time and those who distributed them one at a time (Categories VI and VII). Half of the subjects classified in Category VI started the task by distributing the cars three at a time at each house. It will be recalled that the subjects were required to take three cars on the ferry at a time in the measurement division task of the cargo groups problem. It is possible that some subjects were influenced in some way by the requirements of the first task in choosing to put three cars at a house in the second task. Three five-year-old subjects not included in category VI also distributed three cars at each house but they failed to distribute the remainder correctly. No three-year-olds or eight-year-olds distributed three cars at each house. Half of the subjects who were eight years old made a one by one distribution of the cars. Only five other subjects used this procedure. Six of the older subjects placed five cars immediately at each house. No subjects solved the problem without distributing the cars at the three houses.

Animal Groups

In the partitive division task with the animal groups, the subjects were asked to distribute a set of 18 animals among three cages so that there would be the same number in each cage (see

Appendix A for protocol).

The proportion of successes (N = 18) and failure (N = 12) for this task is comparable to the proportions of success and failure for its counterpart in the cargo groups problem. Each subject who correctly completed the task is classified in one of four categories according to the procedure employed. The same is done for those subjects who were unsuccessful with the task. The following eight procedures were observed.

I. The subject did not attempt the task or the task was abandoned.

II. The subject placed an incorrect number in the first and second cages and then placed the remaining animals in the third cage.

III. The subject placed an unequal number in the three cages and then distributed the remainder incorrectly.

IV. The subject placed five animals in each cage and failed to distribute the remaining three correctly.

V. The subject placed an unequal number in each cage and then distributed the remaining animals correctly.

VI. The subject placed three, four or five animals in each cage systematically, then distributed the rest correctly.

VII. The subject distributed the animals one at a time in the three cages.

VIII. The subject placed six animals at a time in each cage.

The procedures were ordered in what was assumed to represent

a progression from the lowest to the highest level of solution. Table 4 shows the distribution of these procedures according to age and sex.

One can observe from the table that there is wide spread in the range of procedures for the subjects at ages four, five, six and eight, but a very narrow spread at ages three and seven.

Category VI has the greatest frequency and it includes subjects ranging in age from four to eight. Five of the nine subjects classified in this category started the task by placing five animals in each cage and then distributing the remainder evenly. Two other subjects, both five years old, started with the same procedure but they were unable to distribute the remaining three animals correctly. As with the cargo groups, the requirements of the first task may have influenced the subjects in using this procedure in the second task.

Only two subjects distributed the animals one by one in the three cages. This procedure appeared only at ages seven and eight. Two subjects placed six animals at a time in each cage. One subject who used this procedure was four years old and the other was eight. The four-year-old subject counted the animals in each cage in order to verify if each contained the same number. The eight-year-old subject on the other hand said that there were the same number in each cage without having to verify.

Only one three-year-old subject attempted the task and he appeared simply to pick up groups of animals and toss them into the cages. More than half of the four- and five-year-olds were able to arrive at a correct solution. Three subjects in these two age groups

Table 4

Distribution of Procedures for Animal Groups—Partitive Division

Procedure Category	3	4	5	6	7	8	
VIII			F			F	
VII					M	M	Solution N = 18
VI		F	M	M F	M M M F	F	
V		M F	F			M F	
IV			F F				
III				M M			No Solution N = 12
II	F	M		F			
I	M M M F	M					

Age

M - male
F - female

solved the problem while apparently distributing the animals randomly.

Only two six-year-old subjects solved the problem correctly and both distributed the animals systematically. Two subjects in this age group who appeared to be classifying finished with nine animals in one of the cages. All seven-year-olds appeared to be working systematically but two eight-year-olds used what appeared to be an unsystematic procedure. All subjects in these two age groups arrived at a correct solution.

In conclusion, it may be pointed out that half of the subjects used a systematic distribution procedure in their attempt to solve the problem. Only two subjects using such a procedure failed to arrive at a correct solution. Two six-year-old and three eight-year-old subjects attempted to place the animals in the cages according to class. Classification behavior was not identified in the eight procedure categories; it will be referred to later in the section devoted to the discussion of how children reacted to the distractions.

As was indicated earlier, the overall proportions of success and failure in the two partitive division tasks were comparable. A comparison between Tables 3 and 4 reveals, however, that the proportions at ages four and six reverse from one task to the other.

A greater proportion of four-year-olds were successful with the animal groups task (3 out of 5) than with the cargo groups task (2 out of 10). The opposite is true for the six-year-olds. Only two subjects in this age group correctly solved the animal groups task while six were successful with the cargo groups task. The six-year-olds were less successful with the partitive division task for

the animal groups than were the four- and five-year-olds.

Two different procedures involving systematic distribution of objects were observed in both tasks. Some subjects distributed the cars or the animals one at a time while others distributed them several at a time. One by one distribution was more common in the cargo groups problem. Ten subjects distributed the cars one at a time at each house while only two subjects made a one by one distribution in the animal groups task. Only one subject, an eight-year-old boy, systematically distributed the objects one by one in both tasks.

As was indicated earlier, there may have been a relationship between the procedure employed by some children in the second task and the requirements of the first task. Several subjects started the partitive division task by placing three cars at each house or by placing five animals in each cage. It appears that the use of such a procedure was more common in the animal groups problem and was exhibited more often by five- and seven-year-olds. Seven subjects in all used this procedure in the animal groups task and 10 subjects employed it in the cargo groups task.

After having completed the partitive division task in either problem, the subjects were asked if there were the same number of objects in each of the groups that they had made and also how many there were.

None of the three-year-olds had successfully parked five cars at each house. Yet, three subjects in this age group maintained that there were the same number at each house. Four subjects at age four and one subject at age five also indicated that there were the same

number of cars at each house when in fact there were not. Four six-year-old subjects had not partitioned the cars correctly and each one indicated that there were the same number at each house.

A four-year-old subject distributed the cars correctly but failed to state how many cars were at each house. Another subject in the same age range had first parked three cars at each house and then two cars at each house. When asked how many were at each house, without hesitation she showed three fingers on one hand and two fingers on the other hand. All subjects in the five, six, seven and eight age groups who correctly partitioned the cars were able to state that there were five cars at each house.

In response to the questions in the animal groups problem, two four-year-olds, one five-year-old and two six-year-olds who had not made the correct distribution indicated that there were the same number of animals in each cage. A six-year-old boy had placed nine animals in one cage, six in another and three in another. He maintained that there were the same number in each cage even after he counted the animals.

All seven- and eight-year-olds correctly partitioned the animals and all stated that there were six animals in each cage.

In concluding, it should be noted that the younger subjects tended to count the number of objects in each group while the older subjects did not. Up to age six, most of the children counted the objects in each group for the purpose of verification even if they had previously said that there were the same number in each group. Most seven- and eight-year-olds were able to verify the number of

objects by looking at one group only. They appeared to know that the procedure guranteed the other groups would contain the same number as well.

Section 3. Comparison Between Measurement and Partitive Division

A comparison of the solution results for the measurement and the partitive division tasks should reveal whether the subjects in the sample were as successful with tasks involving one division process as with those involving the other.

Cargo Groups

The distribution of correct solutions for the 60 subjects who performed both the measurement and the partitive division tasks with the cargo groups problem is given in Table 5. It reveals that there were a greater number of successes with the partitive division task.

Eleven subjects who were unable to arrive at a correct solution with the measurement division task could do the partitive task. The difference in the number of successes for the two tasks is especially noticeable at age five.

Only two subjects in the sample were unable to solve the second task after successfully solving the first task. One five-year-old and four six-year-old subjects were successful with both tasks of the cargo groups problem. It is only at age seven however that the subjects appear to have control over tasks involving either measurement or partitive division processes.

Table 5

Distribution of Solutions for Cargo Groups Tasks
(N = 60)

	Age 3		Age 4		Age 5		Age 6		Age 7		Age 8	
Subject	M	P	M	P	M	P	M	P	M	P	M	P
S_1						*				*	*	*
S_2						*			*	*	*	*
S_3						*			*	*	*	*
S_4					*	*	*	*	*	*	*	*
S_5								*	*	*	*	*
S_6	*					*		*	*	*	*	*
S_7							*	*		*	*	*
S_8				*			*	*	*	*	*	*
S_9						*	*		*	*	*	*
S_{10}				*			*	*	*	*	*	*

M - Measurement division
P - Partitive division

* - Solution

Animal Groups

A comparison of the number of correct solutions for the two tasks in the animal groups problem reveals that there were almost the same number for each task. Table 6 gives the distribution of solutions for the 30 subjects who attempted the animal groups problem.

Seventeen subjects correctly solved the measurement division task and 18 subjects solved the partitive division task. There were only two subjects who could not do the first task but could do the second.

The three-year-olds seemed to have been unable to cope with the situation while all seven- and eight-year-olds appear to have had no difficulty with either task. Three four-year-old and two five-year-old subjects were successful with both tasks while only one six-year-old was able to accomplish this. The difference in the number of successes for these age groups could be accounted for in part by the size and nature of the sample.

There was a sharp increase in the number of successes from the first to the second task in the cargo groups problem. This pattern does not exist for the animal groups. There is a possibility that the cargo groups measurement division task was structured in such a way as to prevent many subjects from arriving at the solution. The other possibility is that learning may have taken place from the first to the second task.

If the latter possibility is true, it would seem reasonable to assume that learning should also have taken place in the animal groups problem. This argument is supported by the fact that there

Table 6

Distribution of Solutions for Animal Groups Tasks
(N = 30)

	Age 3		Age 4		Age 5		Age 6		Age 7		Age 8	
Subject	M	P	M	P	M	P	M	P	M	P	M	P
S_1						*		*	*	*	*	*
S_2					*				*	*	*	*
S_3			*	*					*	*	*	*
S_4			*	*	*	*	*	*	*	*	*	*
S_5			*	*	*	*			*	*	*	*

M - Measurement division
P - Partitive division

* - Solution

were more similarities in the apparatus for the two animal groups tasks than for the two cargo groups tasks. Since there was no significant increase from the first to the second task with the animal groups, the hypothesis that significant learning took place in the cargo groups problem does not seem to be supported.

Section 4. Behaviors Associated with the Distractions

When mathematical ideas are embodied in a problem-solving situation many aspects of the situation are irrelevant to the concepts and may be considered to be distractions. The two division problems which were designed to facilitate the study of young children's behaviors consisted of models of objects or animals with which the subjects could simulate real world situations. In observing the procedures employed by the subjects as they attempted to solve the problems it became apparent that many children were responding to distractions inherent in the various tasks.

Subjects from each age group interacted freely with the apparatus even though the problems could have been solved with a minimum of interaction. Most subjects went through the motions of taking the cars across on the ferry, parking the cars at the houses or building cages and placing the animals inside. Some subjects appeared to respond so much to certain irrelevant aspects of the tasks that it may have impaired their ability to solve the problems.

Cargo Groups—Measurement Division Task

The apparatus for the measurement division task with the

cargo groups consisted of a set of toy cars of various colors, a ferry boat, and a model of a river with a parking lot on each side. In using the apparatus to solve the given problem, some subjects took great care in placing the cars on the ferry, in docking the ferry and in unloading the cars. Other subjects on the other hand just tossed the cars onto the ferry, took them across the river, and dumped them onto the second parking lot.

Some subjects appeared to choose the cars they were placing on the ferry according to color. Besides being of different color, the cars were also of different makes and there were some indications that subjects were at times choosing cars according to this characteristic. Unfortunately it was not possible to discern color or type of cars on the black and white videotapes. When interpreting these behaviors it should be noted that they were evident only in the few instances when they were accompanied by verbalizations.

Many observable behaviors judged to be responses to the distractions appeared to be simulations of real world situations or experience and are categorized as follows for the cargo measurement division task.

1. The subject pushed (or drove) most of the cars onto the ferry.
2. The subject turned the ferry around during the crossings.
3. The subject docked the ferry perpendicular to the shore at the second parking lot, always at the same spot or always opposite an unoccupied spot, and drove the cars off the ferry.

4. The subject made a motor sound as he or she pushed the cars or the ferry.
5. The subject parked all the cars side by side in one or two rows in the second parking lot.
6. The subject pushed the ferry around an island during the crossings.

There were 22 subjects who exhibited one or more of these behaviors while interacting with the apparatus. The younger subjects responded more to the distractions than the older subjects but some of these behaviors persisted up to age eight. (See Appendix E, Table E-2 for distribution of behaviors.)

Several subjects up to age six exhibited more than one of the categorized behaviors. Nine subjects turned the ferry around while making the crossings (category 2). Since the ferry was concave at one end and convex at the other, the procedure of turning it around usually assured that it would fit with the shore at docking. This was not the case however for a three-year-old girl who systematically turned the ferry at every crossing. The ferry was in such a position that the convex end always touched the shore and there was no fit.

Only younger subjects (ages three to five) made motor sounds to accompany the movement of the cars or the ferry. This behavior was most evident with the three-year-olds. It is interesting to note that although several subjects took great care in docking the ferry and pushing the cars off (category 3), only four subjects systematically drove the cars onto the ferry (category 1). The majority of the subjects lifted the cars onto the ferry. Seven subjects parked

the cars in a row in the second parking lot. All three- and four-year-olds who exhibited this behavior placed the cars side by side all in one row. A five-year-old also parked the cars in this manner. One five- and one six-year-old parked the cars in two rows. Three four-year-old subjects pushed the ferry around the island while making the crossings.

Boys responded more often than girls to the distractions in the task. Almost half of the male subjects exhibited one or more behaviors indicating distraction while only six females out of 26 exhibited such behaviors. Only male subjects systematically drove cars onto the ferry or made motor sounds. Twenty-five subjects correctly solved the problem and only four of these responded in some way to the distractions inherent in the task. It must be pointed out, however, that 17 subjects who did not exhibit semblance of reality behaviors failed to arrive at a correct solution.

Responding to the distractions may have impaired some subjects to the extent that it prevented them from solving the problem. It appears, however, that some other factor or factors could have hindered the subjects' ability to arrive at a correct solution.

Cargo Groups—Partitive Division Task

Those responses to the distractions in the partitive task that could be observed were mostly simulations of real world situations and fall into three categories.

1. The subject systematically pushed, or drove, the cars from the parking lot to the houses.
2. The subject parked the cars on the road rather than on the grass directly at the houses.

3. The subject made motor sounds.

Driving the cars to the houses was the most common of the three behaviors. Twelve subjects were classified in this category. It was also the only one of these behaviors to be exhibited by girls. Three subjects chose to park the cars on the road rather than at the houses. Since the houses were surrounded by grass, the green region of the board, it appears that these subjects would not drive the cars onto the grass. Making motor sounds was mostly exhibited by the younger subjects but this behavior was also observed in one six-year-old.

There were seven subjects at the five- and six-year-old levels who were distracted by some aspect of the task. Only two of these subjects correctly solved the problem. Of the 13 subjects in these two age groups who were not so distracted, 10 were able to arrive at a solution. It appears that being preoccupied by certain reality aspects of the task may have impaired some five- and six-year-old subjects in their ability to solve the problem. It does not appear to have been a major cause of failure at any other age level.

The apparatus for this task consisted of houses of three different colors and cars of four different colors. Some cars were the same color as the houses. A few subjects appear to have associated the color of the cars with the color of the houses and some possibly attempted to group cars of the same color together at a house. As was indicated earlier, color was not discernable on the tapes and behaviors involving classification were only observed in the few instances when verbalizations made it obvious. Such

behaviors, therefore, cannot be systematically analysed for this particular problem. It appears, however, that the younger subjects were more distracted by the colors and were more than prone to matching them than were the older subjects.

Animal Groups—Measurement Division Task

In the first task with the animal groups, most subjects built cages and then proceeded to place the animals inside. The task contained several distractors and the majority of the children responded to these distractions.

The animals were made of plastic or rubber and they could all stand by themselves. Some were painted their natural color and they varied slightly in size according to kind. It will be recalled that the set of animals contained some that were alike and others that were quite different. One of the animals was a lion. It was not possible to solve the problem without grouping certain animals together in cages in a way that would not normally be found in nature.

Observing the children's behaviors revealed that several subjects grouped similar animals together in cages and some even tried to place the lion alone in a cage. The following classes of behaviors were observed.

1. The subject stood the animals up as they were placed in the cage.
2. The subject placed animals of a similar class in the various cages.
3. The subject commented on or talked to the animals.

One three-year-old subject did not attempt the task.

Another subject, in this age group abandoned the task before he built any cages.

All but four subjects who attempted the problem responded in some way to the distraction in the task. Of those who did not respond, one subject was seven years old and two were eight years old. (See Appendix E, Table E-4 for distribution of behaviors.)

The most common behavior among the children was their careful placing of the animals inside the cages. Twenty-two subjects took great care in assuring that the animals were in an upright position. Not only did the majority of the subjects stand the animals, but 14 subjects picked up and stood animals that had fallen. Some would rearrange the animals inside the cages to make room for other animals. Only three subjects (3, 6 and 8 years old) actually let the animals drop into the cages. One seven-year-old and one eight-year-old child solved the problem without placing animals into the cages. One four-year-old boy had a procedure all his own for placing the animals in the cages. Rather than standing the animals, he lifted the slots as if they were gates, passing them under the slots into the cages.

Grouping similar animals together in the same cage was a behavior that was exhibited more often by the older children. Of the twelve subjects who were identified as classifying the animals, nine were six years of age or older. Classification behaviors were not evident at age three but only a few subjects in this age group attempted this task at all.

Comments about the animals were expressed mostly by boys. Only seven subjects in all talked about the animals and two of them

were age six and two were age seven. Comments included those by a four-year-old who said that some animals, like the lion, could bite. A six-year-old built six cages, placed animals in each, but did not want to place the lion in any of them because "it might eat them all." Verbalizations will be discussed in greater detail in the following section.

<u>Animal Groups—Partitive Division Task</u>

More than half of the subjects in the sample responded in some way to the distractions in this task. The response behaviors were observed to be the same as those which have been identified for the measurement division task.

There were 17 subjects in all who took particular care to stand the animals inside the cages while attempting to solve the partitive division task. All five-year-olds again exhibited this behavior. It is interesting to note that this behavior was displayed by most eight-year-olds. Behaviors involving classification appear only among the older subjects. Seven subjects who had attempted to classify the animals in the first task did not do so in the second task. An eight-year-old classified in the second task only. Only three subjects made comments pertaining to the animals while attempting to solve the partitive division task. Two of these subjects were six years old and one was five.

It cannot be concluded from the data that responses to the distractions impaired the subjects' ability to solve the task. It should be noted, however, that none of the younger subjects appeared to have classified the animals and two six-year-olds who attempted to

group similar animals together failed to solve the problem.

Section 5. Verbalizations

The analysis of the data has thus far revealed that the majority of the subjects interacted freely with the apparatus. Many of the children in the sample made spontaneous verbalizations while attempting to solve the problems. Some of these verbalizations, such as counting, appeared to be closely related to the tasks while others appeared more closely related to the apparatus. It will be recalled that the subjects were asked questions pertaining to the problems. The responses to such questions are not analysed in this section; only the subjects' spontaneous verbalizations are discussed.

Table 7 gives the number of subjects in each age group who made some form of verbalization during the cargo and the animal group tasks. The table reveals that the majority of the subjects

Table 7

Number of Subjects Verbalizing

	Age Level					
Task	3	4	5	6	7	8
Cargo groups problem (N = 60)	7	6	7	7	5	6
Animal groups problem (N = 30)	3	4	4	5	2	5

spoke while attempting to solve the problems. It also shows that more subjects at the six-year-old level made verbalizations than at any other age level.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

It is seen from the table that a greater proportion of subjects (23 out of 30) verbalized during the animal groups problem than during the cargo groups problem (38 out of 60). Of the 30 subjects who performed the cargo groups problem only, eight subjects did not verbalize. Twenty-nine subjects attempted both the cargo and the animal groups problems and only four of them made no verbalizations. It will be recalled that a three-year-old boy did not attempt the tasks.

All the observed verbalizations in the cargo and the animal groups problems are classified into seven categories. They are the following.

a. Asking for clarification of rules.

b. Monitoring the actions.

c. Rule making.

d. Comments pertaining to the apparatus.

e. Comments pertaining to the task.

f. Counting.

g. Indicating completion of a task, or part of a task.

Examples of the different categories reveal the type of verbalizations the children made while they were engaged in the tasks. Some subjects asked questions pertaining to a task. In asking for clarification in the first part of the cargo groups problem, the children asked questions like "Okay, right now?", "That makes three?", "Again?". In the second task, several of the younger subjects parked one car at each of the three houses and then asked questions like "Now where shall I put the other ones?" or "Where all these cars go?". Some five- and six-year-olds asked the following questions:

"Three again?", "Have to be the same color?", "Okay, but how do I get them off the grass?". In the animal groups problem, the subjects asked such questions as "Should I build another one?", "Do they have to be the same animals?", "Five again?".

The measurement division task with the cargo groups elicited much monitoring of the actions from the younger children. As the children moved the cars or the ferry around, they made such comments as "Back up, park it," "Now it's coming back," "Off the boat." There was less monitoring in the animal groups problem, but there were a few comments like "Put an elephant here," "Put another one in." Comments classified as rule making included "Three, could take four!", "I think I'll take two this time," "The yellow car wants to go back," "Supposed to be the duck cage."

The animal groups problem generated many comments about the apparatus, especially the animals. Subjects made such comments as "Tiger's bigger, bite," "I don't like lions, do you?", "His hands are locked," "They need the door to get out." Comments on the tasks included remarks like "That's going to be a little easier than the other one," "Gonna be too many cages," "I know what I'll do." Some subjects made comments to indicate that they had completed a task. The most common expression was "There!". Other comments included "That one done," "No more cars."

Table 8 shows the frequency for the seven categories of verbalizations. It reveals that verbalizations involving clarification of rules occurred more often than any other form of verbalizations. There was a significant increase from task 1 to task 2 in the

cargo groups problem. The next highest frequency for a verbalization category was category d. Comments pertaining to the apparatus were

Table 8

Frequences for the Seven Verbalization Categories

Problem		Verbalization Category a	b	c	d	e	f	g
Cargo Groups N = 60	task 1	7	9	8	7	5	6	5
	task 2	12	2	6	8	10	2	4
Animal Groups N = 30	task 1	6	3	4	11	5	4	3
	task 2	6	0	1	3	5	5	2
Total		31	14	19	29	25	17	14

most common in task 1 of the animal groups problem.

It is interesting to note that the children had a tendency to monitor their actions more in the measurement division task with the cargo groups than in any other task. This was the first of the four tasks attempted and the one that required the greatest number of moves in order to complete. Overt counting did not occur as often as one might expect in this type of mathematical task. A few subjects did count the cars as they placed them on the ferry or they counted the animals as they placed them inside the cages. Only one subject, a six-year-old, counted overtly the number of trips the ferry was making. Hand and lip movements exhibited by the subjects during the various tasks seemed to indicate that some may have been doing covert counting.

The data reveal that the majority of the subjects not only interacted freely with the apparatus but that they also felt at ease while doing so. This is evident by their many spontaneous verbalizations while attempting to solve the various tasks.

The results of the investigation show that most subjects exhibited some form of free verbalization. The animal groups problem appears to have elicited more verbalizations than the cargo groups problem. It was noted that a greater number of subjects at the six-year-old level vocalized than in any other age group. The frequency of verbalizations in each age range reveals, however, that the three-year-olds were the most vocal while the seven-year-olds were the least vocal.

Asking for clarification of rules was the most common form of verbalization and was expressed most often by girls in task 2 of the cargo groups problem. The next highest frequency for a verbalization category was for comments pertaining to the apparatus. These comments were made more often by boys in task 1 of the animal groups problem. The three- and four-year-old subjects had a tendency to monitor their actions more than the older subjects and they exhibited this behavior more often in task 1 of the cargo groups problem. Relatively few subjects were observed to count.

PART II. LONGITUDINAL SAMPLING

The theoretical distribution of the problem sets for the longitudinal sampling provided for five subjects at each age level to be given the cargo groups problem and 10 subjects in each age group to be confronted with the animal groups problem. Because 16 of the original subjects did not participate in this phase of the investigation the number of subjects who attempted the problems in 1975 was reduced in every age group except at age seven. (See Appendix D.) In order to check the validity of the procedures and behaviors identified in the analysis of the cross-sectional data, the analysis of the behaviors for the longitudinal data is treated in the same manner.

Section 1. Measurement Division

Cargo Groups

Twenty-one of the 60 subjects who attempted the cargo groups problem in 1974 were again given the problem in the longitudinal sampling. The procedures employed by the subjects in attempting to solve the measurement division task were observed to be the same as those used in the cross-sectional sampling. These procedures are therefore classified according to the same categories.

A slight change in the protocol for the longitudinal sampling required a change in category III. Whereas subjects could be prompted more than once in the cross-sectional interview when they placed more than three cars on the ferry (1974), they could be corrected only once in the longitudinal sampling (1975). A subject who may have

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

been classified in category III in the cross-sectional sampling could now be classified in category II because he would not be reminded a second time of the number of cars to be placed on the ferry each trip. Procedure category III for this phase of the study is as follows:

> III. The subject placed an incorrect number of cars on the ferry for one trip, was prompted, rectified but failed to arrive at a solution.

All other categories remain the same (see p. 33). The distributions of the procedures employed by the subjects in the cross-sectional and in the longitudinal samplings are given in Table 9. Although fewer subjects attempted the task in 1975 the overall distribution remains remarkably unchanged. The proportion of solution/no solution has increased but this is to be expected as there are fewer younger subjects and more older subjects. The proportions at age six and seven have changed slightly. The six-year-olds appear to have more success than the subjects of the same age in the cross-sectional sampling whereas a smaller proportion of seven-year-olds arrive at a solution. A greater proportion of seven-year-olds are also in lower categories than their counterpart in 1974. This appears to be an artifact of the sample.

Animal Groups

Of the 44 children who attempted the animal groups problem in the longitudinal sampling, 21 had been given the problem in the cross-sectional sampling. (See Appendix D.)

In order that the structure of the protocol for the measurement division task in the animal groups problem be as close as

Table 9

Distribution of Procedures for Cargo Groups:
Measurement Division

Cross-sectional

Procedure Category	3	4	5	6	7	8	
VIII					M M M	F F F	Solution N = 25
VII				F	M M M F	F F	
VI					M	M	
V	F		F	M M F F		M M F F	
IV	F	M M M F F	M M F F F F	M M	M M		No Solution N = 35
III	M M	M M M F	M F F	M M			
II	F F F	M		M			
I	M M M						

Age

M - male
F - female

Longitudinal

Procedure Category	4	5	6	7	8	9	
VIII			F		M	F	Solution N = 13
VII			F	F	M	M F	
VI							
V			F	M F	M M		
IV	F	M M	F	M			No Solution N = 8
III				M			
II	M	F					
I							

Age

M - male
F - female

possible to that of the corresponding task in the cargo groups problem, a question was added for the longitudinal interviews. If a child placed other than five animals in each cage he was asked: "How many animals were to go in each cage?" The addition of this question to the protocol does not necessitate any change in the categories used to classify the procedures employed by the subjects. The categories established for the cross-sectional sampling serve to classify the procedures observed in the longitudinal data (see p. 38 for categories I to VIII). The distributions of the procedures employed in both samplings are given in Table 10.

A ninth category, namely

IX. The subject looked at the apparatus and gave the solution without manipulating the material,

had to be added for the longitudinal data because several subjects solved the problem in this manner. The greatest number of subjects choosing this procedure appears at age nine but it was also used by some six-, seven- and eight-year-olds. Although this method of solving the problem does not show up in the cross-sectional sampling, it must be remembered that there were more subjects in each age group, except at ages four and five, attempting the problem in 1975.

Here again the proportion of solution/no solution is greater for the longitudinal sampling than it is for the cross-sectional sampling. It is true that there are no three-year-olds in the longitudinal sample, and subjects in this age group had not had success with the problem in 1974, but this situation alone does not account for the difference in proportions. The greatest difference between

Table 10

Distribution of Procedures for Animal Groups:
Measurement Division

Cross-sectional

Procedure Category	3	4	5	6	7	8	
VIII					M	F	Solution N = 17
VII		F	F			M F	
VI		F			M M	M F	
V		M	F F	F	M M		
IV	M			M F			No Solution N = 13
III		M	F	M			
II	F	M	M	M			
I	M M F						

Age

M - male
F - female

Longitudinal

Procedure Category	4	5	6	7	8	9	
IX			F	M F	M M	M M F F	Solution N = 39
VIII			M	M	M M M M M	M F	
VII							
VI			M F F	M F F	M M		
V	M F	M M M F	F F F F	M M M			
IV							No Solution N = 5
III	F F						
II		M		M			
I	M						

Age

M - male
F - female

the two samplings appears at age six. Only one subject correctly solved the problem in the first sampling while all nine subjects arrived at the solution in the second sampling. The six-year-olds did not appear to perform as well as the four- and five-year-olds in this task during the cross-sectional interviews. This finding is not substantiated by the longitudinal data. The pattern of responses suggests that the differences are artifacts of the sampling.

The protocol for the longitudinal interviews permitted the examiner to question the subjects if more than five animals were placed in one cage. Eight subjects attempted to place six animals in a cage and were either reminded that only five were to be placed in a cage or they were asked: "How many animals were supposed to go in each cage?" All these subjects then proceeded to remove one animal from the cage. Of these eight subjects, two were age four, three were age five, and three age seven. Two five-year-olds and two seven-year-olds correctly solved the problem after being prompted in this manner.

It should be possible in the longitudinal aspect of the study to note the apparent growth of a child in certain areas of problem solving after the period of a year. Eight of the 21 subjects attempting the cargo measurement task for a second time did not change their procedure from the previous year. Nine of them used a more sophisticated procedure for completing the task in the second year and three actually used a more immature procedure to deal with the task. (See Table E-6 in Appendix E for distribution of procedures employed by individual subjects.)

Comparing procedures employed by the subjects while attempting the animal measurement division task in both years shows that five subjects used the same procedure both times. Twelve subjects used a more sophisticated procedure in the longitudinal interview while four used a more immature procedure. A new procedure was used by subjects in the six to nine year age range in the longitudinal sampling. This procedure involved no manipulation of the objects and a similar procedure had been observed in the cross-sectional data with the cargo groups.

Section 2. Partitive Division

Cargo Groups

The number of subjects attempting the second task of the cargo groups problem for the longitudinal sampling is relatively small compared to the number for the previous year. The behavior of age groups will be compared as will the change in behavior of individuals from the cross-sectional to the longitudinal sessions.

It will be recalled that all the subjects in the longitudinal sample had attempted the task the previous year except for a four-year-old who had abandoned the problem. In viewing the tapes for this part of the study it was noticed that some subjects employed procedures that had not been observed previously. Two new categories therefore had to be added to the eight of the cross-sectional analysis. (See p. 42 for categories I to VIII.) The two procedures added are:

IX. The subject made three groups of five cars without placing them at the houses and gave the correct solution.

X. The subject looked at the apparatus and gave the answer without manipulating the material.

Table 11 gives the distribution of the procedures employed by the subjects in both samplings. One seven-year-old subject is not included in the classification for the longitudinal analysis because a protocol error rendered the task a non-problem. In both distributions procedures designated as of a higher level became more frequent as the age level increased. Three subjects in the longitudinal sampling were able to solve the problem without any manipulation of the material. This procedure may have been available to some subjects the previous year but none chose to use it. It is possible that the novelty of the situation may have prompted procedures involving manipulation to be employed the first time.

The noticeable difference between the two distributions is the proportions of solution/no solution at ages six and seven. In the cross-sectional distribution, all ten subjects aged seven are classified in the solution categories. Although there were only four subjects at this age level in the longitudinal sample, two failed to arrive at a correct solution. These subjects also did not use a systematic procedure in attempting to solve the problem. All six-year-olds in the longitudinal sample solved the problem while several in this age group had failed to do so in 1974. This observation lends further support to the suggestion made previously that the behaviors of the six-year-old group in the cross-sectional session and that of those who remained in the longitudinal sample were not typical for that age group.

Table 11

Distribution of Procedures for Cargo Groups:
Partitive Division

Cross-sectional

Procedure Category	3	4	5	6	7	8	
VIII					M M F	M F F	Solution N = 34
VII			F	M M F	M	M F F F F	
VI		F	M F	M M F	M M M M	M F	
V		F	M M F		M M		
IV	F F	M M M F	F F F F	M F			No Solution N = 26
III	M F	M M		M			
II	M			M			
I	M M M F F	M M					

Age

M - male
F - female

Longitudinal

Procedure Category	4	5	6	7	8	9	
X					M	F F	Solution N = 15
IX				M	M	M	
VIII				M			
VII							
VI		M F	F F		M M		
V			F F				
IV							No Solution N = 5
III	F	M		M F			
II							
I	M						

Age

M - male
F - female

<u>Animal Groups</u>

Procedures that were not evident in the previous year's data were again observed in the longitudinal sampling for the partitive division task of the animal groups problem. Three new categories are added to those of the cross-sectional analysis. They are:

IX. The subject made three groups of six animals on the table and gave the correct solution.

X. The subject placed some animals into the cages and gave the solution without placing remaining animals in cages.

XI. The subject looked at the apparatus and gave the correct solution without manipulating the material associated with the problem.

Other procedures employed by some subjects necessitated changes in category IV. This category now reads:

IV. The subject placed three, four or five animals in each cage and failed to distribute the remaining animals correctly.

The other categories remain unchanged and may be found on page 46.

Table 12 shows the distributions of the procedures employed by the subjects during the cross-sectional and the longitudinal interviews. The number of correct solutions in the longitudinal classification far exceeds the number of no solutions and several of the older subjects are classified in the upper categories. Although the procedures described by categories IX, X and XI did not occur in the cross-sectional data, it must be remembered that there were more

Table 12

Distribution of Procedures for Animal Groups:
Partitive Division

Cross-sectional

Procedure Category	3	4	5	6	7	8	
VIII			F			F	Solution N = 18
VII					M	M	
VI		F	M	M F	M M M F	F	
V		M F	F			M F	
IV			F F				No Solution N = 12
III				M M			
II	F	M		F			
I	M M M F	M					

Age

M - male
F - female

Longitudinal

Procedure Category	4	5	6	7	8	9	
XI					M M M	F	Solution N = 32
X					M	M F	
IX					M M	M F	
VIII	F		M F	M F	M		
VII		M		M M	M		
VI		M	FF FF FF	F	M	M	
V				M			
IV				M M F			No Solution N = 12
III	M						
II	F F	M	M	M			
I	M	M F					

Age

M - male
F - female

subjects per age group for this problem in the longitudinal sample.

A comparison of the two distributions again reveals a difference in solution/no solution proportions for the six- and seven-year-olds. Only one out of nine six-year-olds failed to solve the problem in the longitudinal sampling while three out of five failed to arrive at a solution in the cross-sectional sampling. The reverse is true at age seven. All the subjects in this age group had solved the problem the first year and four out of ten failed to do so in 1975.

It was noted earlier that the six-year-old subjects were less successful than the four- and five-year-olds with both animal groups tasks in the cross-sectional sampling. These same subjects, who were seven years of age at the time of the longitudinal data collection, performed at a lower level that the six-year-olds in the same sampling. This appears to be an artifact of the sample rather than the age level.

Analysis of the cross-sectional data for the partitive division tasks revealed that the younger subjects had a tendency to state that subsets contained the same number of objects even if they did not. This behavior is still evident in the longitudinal sampling although it is less frequent. A four- and a five-year-old subject failed to partition the cars equally and yet they indicated that there were the same number at each house. Five subjects (three four-year-olds and two seven-year-olds) maintained that the three cages of the animal groups task contained the same number of objects after they had partitioned them unequally.

Younger subjects in the cross-sectional sampling tended to count the number of objects in each group when verifying whereas many of the older subjects counted the number in one group only. Counting the number of objects in each group is still evident in the five- and six-year-old age groups of the longitudinal sample but it does not appear to be as frequent. One nine-year-old also exhibited this behavior. Most of the older subjects gave the correct number in the groups without any evidence of counting or after a brief look at the cages.

It was suggested in the cross-sectional analysis that the requirements of the first task may have influenced some subjects' choice of procedure for the second task in both problems. Although evidence of this still shows up in the longitudinal sampling, procedures that support this contention are not as frequent as they were in the previous sampling. Only one subject started the cargo group partitive division task by placing three cars at each house. Seven subjects started the equivalent task in the animal groups problem by placing five animals in each cage. A five- and a seven-year-old who used this procedure failed to arrive at a solution.

It was also pointed out in the cross-sectional analysis that one by one distribution in the partitive division task was not as common as one would expect. This is borne out by the longitudinal data. No subjects distributed the cars one by one to the houses and only four subjects systematically distributed the objects in this manner in the animal groups problem (category VII).

Comparing procedures employed by individual subjects for the

cargo partitive division task in both samplings shows that 13 children employed a higher level of solution when they were a year older. Three subjects used the same procedure and four used a more immature procedure. (See Appendix E, Table E-6, for distribution of procedures.)

In attempting this task in the longitudinal interviews, some subjects in the seven to nine age range used procedures that had not been observed in the data for the previous year. Therefore new categories had to be added. These procedures, however, involved strategies that had been employed in the cargo measurement division task during the cross-sectional interviews.

All but three of the 21 subjects who were given the animal partitive division task for a second time used a more sophisticated procedure in the longitudinal sampling. Only one subject used the same procedure both times. Procedures that had not been used in the cross-sectional sampling in the animal partitive task were observed in the longitudinal data. These new procedures were employed only by the eight- and nine-year-olds. Two of these procedures involved strategies that were used with the animal measurement division task and another was employed with the cargo measurement task by subjects in the cross-sectional sample.

Section 3. Comparison between Measurement and Partitive Division

The analysis of the cross-sectional data revealed that the younger subjects had more success with the animal groups problem than with the cargo groups problem. It was also shown that more

subjects correctly solved the partitive division task of the cargo groups problem than its measurement division task. No meaningful differences were observed between the two animal groups tasks.

Of the 20 subjects who attempted the cargo groups measurement and partitive tasks in the longitudinal session two four-year-olds failed to arrive at a solution for either. The eight- and nine-year-olds and five younger subjects were able to do both. Three children could do the partitive division but not the measurement division task. Only one child who did the measurement task could not do the partitive task. (See Appendix E, Table E-7, for solution distribution.) The evidence here is not convincing in itself but when combined with the cross-sectional data tends to support the contention that the partitive division task was easier than the measurement division task in the cargo groups problem.

In the animal groups problem the measurement division task gave less trouble. Of the 44 subjects in the longitudinal sample doing the animal groups problem 30 were able to do both measurement and partitive tasks correctly. Two children did the partitive task but could not do the measurement one. Nine children did the measurement task but not the partitive one. Two four-year-olds and a six-year-old could do neither. (See Table E-8, Appendix E, for distribution of solutions.)

These findings seem to indicate that the partitive division task was more difficult than the measurement division task with the animal groups. This is supported by the fact that the structure of the two tasks had many similarities and that the partitive division

was attempted last. Furthermore, the subjects did not have to overcome the distraction of building cages in the second task as these were already built for them.

Twenty-one subjects attempted both the cargo and animal groups problems in the longitudinal sampling. A comparison of the number of solutions for the two measurement division tasks reveals that five subjects correctly solved the animal groups task only but no subjects correctly solved the cargo groups task only.

Of the 30 subjects who attempted both measurement division tasks in the cross-sectional sampling, seven correctly solved the animal groups task after failing to solve the cargo groups task. Only one subject solved the latter and failed to solve the former. It will be recalled that the subjects attempted the cargo groups problem before the animal groups problem in the first sampling. It could be hypothesized from the cross-sectional data that the increase in the number of solutions for the animal groups task was accounted for either by learning or by the structure of the tasks, or by both. When the order of presentation was reversed in the longitudinal sampling, there were still a greater number of successes for the animal groups task. It appears, therefore, that the animal groups measurement division task was easier for the subjects to solve than the equivalent task with the cargo groups.

Comparing solution results in the longitudinal sampling for the 20 subjects who attempted the two partitive division tasks reveals that three subjects solved the animal groups task only and two subjects solved the cargo groups task only. The tasks appear,

therefore, to have been of about equal difficulty.

The distribution of solution results for the 20 subjects who attempted the cargo and animal groups problems in both samplings is given in Table E-9 of Appendix E.

Section 4. Behavior Associated with the Distractions

Distractions inherent in the various tasks are identified in the first part of this chapter. The behaviors associated with these distractions are classified on pages 57, 59, and 61.

Cargo Groups Problem

Behaviors identified as responses to the distractions were exhibited mostly by five- and seven-year-olds. Two boys, age five, systematically pushed the cars onto the ferry, turned the ferry around during the crossings, and drove the cars off the ferry. One of these subjects appeared to choose several of the cars that he placed on the ferry and he also pushed the ferry around the island several times. These two subjects also pushed the cars to the houses in the second task. Neither subject correctly solved the measurement division task and only one solved the second task. Two seven-year-olds, a boy and a girl, appeared to choose some cars that they were placing on the ferry. The boy also pushed the ferry around the island several times and drove the cars off. One subject, age four, made motor sounds and a seven-year-old boy made a horn sound.

Classification behaviors were exhibited by two seven-year-old subjects in the partitive division task. When one of these subjects was given the instruction for the task, he commented, "I

was figuring on putting all the yellow at one house." The interviewer made a protocol error and indicated that there should be five cars at each house. The subject distributed the cars equally, placing some cars on the road opposite the houses, but because of the protocol error his solution was not included in most of the analysis for this task. Another seven-year-old boy who did much verbalizing during the task indicated that he was placing cars of a certain color at houses with the same color. He completed the distribution with one car at one house, five at another and nine at another.

The hypothesis that the distractions could have prevented certain subjects from solving the cargo groups tasks is supported by the evidence of the longitudinal study.

Animal Groups Problem

As was indicated earlier, the variety of animals was an inherent distraction in the animal groups problem. Behaviors identified as responses to these distractions consisted of standing the animals, classifying them, and making comments about them.

The longitudinal data reveal that 19 subjects were distracted in both animal groups tasks. Six other subjects displayed such behaviors in one task only, three in the measurement division task and three in the partitive division task. The data show that standing the animals up inside the cages was still the most common of these behaviors. Twenty-two subjects insisted on standing the animals up. Two seven-year-olds were among those who failed to solve the problem and who seemed to be unduly distracted by the situation.

In order that classification behaviors be more easily

identified in the longitudinal recordings, cages were often dismantled after each task and the camera focused on the animals. Although it was not always possible to determine if the animals had been grouped according to some classification scheme, some arrangements revealed groupings that had not been observed in the cross-sectional data. Some subjects appeared to classify according to size. Some cages contained mostly small animals and others the larger animals. A seven-year-old boy appeared to classify in order of increasing size. At the end of the partitive division task, the animals in each cage were standing one behind the other in line, the smaller animals in front and the larger behind. In the measurement division task, this subject had also lined up the animals in the cages but they were in reverse order in some of the cages. This subject correctly solved both animal groups tasks.

No four-year-olds in the longitudinal sample chose animals according to class. This behavior was exhibited by more five-year-olds in the sample than in the first sample. The reverse is true at age eight. Several eight-year-old subjects had classified the animals in the cross-sectional sampling. Although there were more subjects in this age group in the longitudinal sample, only one exhibited classification behaviors. However, five subjects chose to solve the partitive division task without placing animals into cages and two solved the first task in this manner.

Eleven subjects classified the animals in the measurement division task and nine of these correctly solved the problem. Eight subjects grouped the animals according to some classification scheme

in the partitive division task and five arrived at a solution. Classifying the animals does not appear to be a major factor in preventing the children from solving the tasks correctly. None of the subjects included in this distribution exhibited this behavior alone. Nine subjects also stood the animals in the measurement division task and seven did so in the partitive division task. The three other subjects also commented on the animals. (See Table E-10, Appendix E, for distribution of behaviors.)

Although more children attempted the animal groups problem in the longitudinal than in the cross-sectional sampling, fewer subjects made comments on the animals. Four subjects made comments while attempting the measurement division task. These included a four-year-old boy who was preoccupied with building high fences so the animals would not jump out or to prevent the "pigs" from getting into the horses' cage and having a "fight up." He also commented about other animals having "sharp teeth." During the second task comments were made by a five-year-old girl who appeared to be carefully selecting animals throughout the task and by a seven-year-old boy who said that one of the animals was looking for his friend. Of the six subjects who commented on the animals, only the two girls who made comments during the measurement division task arrived at a correct solution. A few subjects commented on the animals between tasks.

Section 5. Verbalizations

Analysis of the cross-sectional data has revealed that the subjects exhibited many spontaneous verbalizations while attempting to solve the various tasks. Verbalizations are again studied in the longitudinal data in order to compare the types and the frequencies with those of the first sampling.

Examination of the data reveals that the proportion of subjects exhibiting verbalizations during the longitudinal data collection is not as great as it was in the first sampling. Slightly more than half of the subjects made comments while attempting the problems in 1975. There appears to be no difference in proportion of verbalizations/no verbalizations for the cargo and animal groups problems.

In the longitudinal sampling the six-year-olds did not exhibit many verbalizations while the seven-year-olds did. The frequency of verbalizations expressed by the subjects in these age groups does not appear to depend as much on age as on the sample.

Table 13 gives the frequencies for the seven categories of verbalizations exhibited by the subjects in the longitudinal sample. It reveals findings similar to those of the cross-sectional data. Asking for clarification of rules was still the most common form of verbalization, followed closely by comments pertaining to the apparatus. Task 1 of the animal groups problem still elicited the most comments about the apparatus. There was a decrease from the cross-sectional data in the number of subjects expressing comments classified as rule making. This decrease can be explained in part by the fact that several three-year-olds, in the first sample, had made this type of comment and there are no longer any subjects in this age

[illegible]

Table 13

Frequences for the Verbalization Categories

Problem		Verbalization Category a	b	c	d	e	f	g
Cargo Groups N = 21	task 1	1	3	1	3	1	1	2
	task 2	3	3	4	2	2	2	0
Animal Groups N = 44	task 1	8	1	4	9	3	5	0
	task 2	6	0	0	2	6	4	0
Total		18	7	9	16	12	12	2

a - Asking for clarification
b - Monitoring the action
c - Rule making
d - Comment pertaining to apparatus
e - Comment pertaining to task
f - Counting
g - Indicating completion of task

range in the longitudinal sample. Furthermore, only two four- and three five-year-olds attempted this problem in 1975.

There was a surprising decrease in the number of expressions indicating completion of a task. Such comments may not be as common among young children as the cross-sectional data had suggested. There is also the possibility that when the children attempted the tasks the first time, the laboratory set-up, which was unfamiliar, may have prompted them to exhibit this behavior more often.

Subjects monitored their actions most often when they were solving the cargo groups problem. It appears that the frequency of such comments depended a great deal on the structure of the task.

Questions about rule clarification were expressed by only a few of the younger subjects and the frequency increased with age. Only four of the older subjects asked for clarification in the cargo groups problem. A four- and a six-year-old asked such questions during task 2 of the animal groups problem but it is at age seven that the frequency increases. There appears to be a tendency for the younger children to ask more questions in the animal partitive division task and for the older subjects to do so in the measurement division task.

Requests for clarification had appeared at every age level in the cross-sectional sampling and were more frequent in the cargo partitive division task. Several of the younger subjects, mostly three-year-olds, had asked where they could park the remaining cars after they had distributed one at each house. This behavior was not

manifested in the longitudinal data.

As had been observed in the earlier data, overt counting is not too evident in the longitudinal data. It appears mostly among the younger subjects in the animal groups problem. Two eight-year-olds also exhibited this behavior. Evidence that many subjects were doing some covert counting was often revealed by the movement of their lips, the action of their hands, or their indication that they had done so.

All verbalizations examined were exhibited by the subjects while attempting to solve the problems. Some verbalizations were expressed by subjects before the problem was stated or after completion of a task. As the cars were being placed on the table, a four-year-old girl commented: "I saw this one before." She also asked: "Now, how do they get across the river?" After the interviewer's instructions, she said: "I know how they go, cause I found out last year." This subject failed to solve either of the cargo groups tasks. After having the apparatus explained to her, a six-year-old girl commented: "Oh, now I remember." This subject solved the measurement division task by making five groups on the table. She also succeeded in distributing the cars correctly in the second task. No other subjects indicated that they remembered the problems from the previous year.

Experimenter Reliability

The experimenter's observations were compared with those of two independent observers in order to establish some estimate of reliability. The data chosen for this purpose were the animal groups

problem in the longitudinal sampling. The reliability check was performed by two graduate students.

One subject from each age group was randomly chosen and the judges viewed the tapes independently. The tape for one subject not included in the reliability check was first viewed by each observer and the recording sheet compared with that of the experimenter in order to establish proper use of the coding system. After areas of agreement and disagreement were identified, the judges viewed the tapes for the six subjects that had been chosen.

Arrington's method of calculating the coefficient of agreement between observers was used (Bott, 1933, p. 67). Table 14 gives the percentage of agreement between experimenter and observers for each subject. A mean coefficient of agreement of .907 was established.

Concluding Statements

The findings from the cross-sectional and the longitudinal phases of the investigation were reported in this chapter. The longitudinal data support many of the cross-sectional findings. Most of the procedures employed by the subjects in the first sampling were again observed in the second sampling. A few new procedures which involved a minimum of interaction were identified in the latter. The cross-sectional data revealed that the six-year-olds were not as successful as some of the younger subjects with some of the tasks. The longitudinal data seem to indicate that this was a characteristic of the sample rather than the age range.

The following general statements summarize the specific

Table 14

Experimenter-Observer Coefficients of Agreement for Animal Groups Problem

Subject	Experimenter vs Observer 1	Experimenter vs Observer 2
S_1	.845	.906
S_2	.845	.903
S_3	.884	.918
S_4	.947	.945
S_5	.934	.928
S_6	.909	.916

Mean Coefficient of Agreement = .907.

findings which are considered important.

1. Children were more successful with the measurement division task associated with the animal groups than with that associated with the cargo groups.

2. The structure of the two animal groups tasks had many similarities. However the subjects had less success with the partitive division task.

3. The various tasks contained distractions to which many of the subjects responded.

4. The majority of the subjects used a systematic procedure in their attempt to solve the partitive division problems. Some procedures, however, may have been influenced by the requirements of the first task.

5. The younger subjects had a tendency to state that the groups that they had made were equal when they were not or when the partitioning had been done correctly they tended to count the objects in each group even after stating that each contained the same number.

6. The children's procedures changed and became more sophisticated as they got older.

7. The children displayed a variety of verbalization behaviors while attempting the problems.

CHAPTER IV

SUMMARY, DISCUSSION AND IMPLICATIONS OF RESULTS, CONCLUDING STATEMENTS, RECOMMENDATIONS FOR FURTHER RESEARCH

SUMMARY

The present study on young children's problem-solving behaviors forms part of a larger investigation undertaken by Nelson and Sawada (1974) to obtain information on how children go about solving problems. Much research on problem solving has been reported but few of the studies conducted were concerned with young children's behavior. Little is known at present on how to devise appropriate problem-solving situations for children. Such information must first be obtained if problem solving is to be included in a model for mathematics learning. It is hoped that studying children's behavior while attempting to solve problems will generate guide-lines for designing problem-solving situations.

Working from the theoretical framework of Piaget, Bruner and Dienes, Nelson and Sawada (1974) designed 12 problem-solving situations to investigate nonverbal problem-solving behaviors of young children. These problem situations attempted to incorporate the characteristics of 'good' problems, as outlined by Nelson and Kirkpatrick (1975).

The Nelson and Sawada investigation consisted of a clinical study and was comprised of a cross-sectional and a longitudinal sampling. The first interviews were conducted in the summer of 1974 and the sample was composed of 90 children ranging in age from three

to eight. The longitudinal sampling was conducted the following summer and the sample consisted of 76 of the original subjects who were available.

The aim of the present study was to discover how the children would behave while attempting to solve two of the 12 problem-solving situations devised for the Nelson and Sawada (1974) investigation. The specific purposes of the study were:

1. To encode the behaviors exhibited by the subjects while they were engaged in solving
   (i) the cargo groups problem,
   (ii) the animal groups problem.
2. To catalogue and analyse these behaviors.

Sixty subjects, 10 at each age level from three to eight, attempted the cargo groups problem during the cross-sectional interviews and 30 of these subjects, five at each age level, were also presented the animal groups problem. In the longitudinal sampling, 21 subjects were given the cargo groups problem and 44 subjects attempted the animal groups problem. Each problem consisted of a measurement division task and a partitive division task.

A descriptive analysis of the children's interaction with the apparatus while attempting the problems revealed a wide range of behaviors. Examination of the procedures employed by the subjects showed that some went through the motions of taking cars across on the ferry, driving cars to the houses, building cages and standing the animals inside, while others simply made groups on the table or gave the solution without any manipulation. The data also revealed

that the problem situations contained distractions to which many subjects responded. Many verbalizations were also expressed by the subjects while they attempted to solve the various tasks.

The findings revealed that the younger subjects had difficulty coping with some of the situations. Very few three-year-olds were able to solve any of the problems attempted. Only a few four- and five-year-olds were able to solve the cargo measurement division task. This problem appeared to be more difficult to solve than either the cargo partitive or the animal measurement division task. The longitudinal data revealed that fewer subjects solved the animal partitive division task than the measurement division task for this problem.

The data showed that the requirements of the first task of a problem often appeared to influence some subjects' choice of procedure for the second task. Although many subjects employed a systematic procedure in attempting to solve the partitive division tasks only a few used a one by one distribution. Subjects up to age six or seven had a tendency to state that the objects were partitioned equally even if they were not. When verifying the number of objects in each group, subjects up to age six had a tendency to count the objects in each even if they had previously indicated that each group contained the same quantity.

Responses to the distractions inherent in the various tasks included classification behaviors and making detailed simulations of real world situations. Although classifying could not be observed and analysed systematically, certain behaviors were identified. Some subjects grouped cars of the same color together and others associated

the color of the cars with those of the houses. Classification behaviors could more easily be observed in the animal groups problem. Some subjects grouped animals according to kind and others according to size.

Responses to the distractions in the cargo groups problem consisted of driving the cars onto and off the ferry, turning the boat so it would fit with the shore, parking cars on the road, making motor sounds, and the like. Such behaviors were exhibited mostly by boys and they persisted up to age six. Behaviors associated with the distractions in the animal groups problem included standing the animals inside the cages. The majority of the subjects exhibited this behavior though it did not appear to interfere with the ability of most to solve the problems.

Examination of the data revealed that many subjects exhibited spontaneous verbalizations while attempting the various tasks. These verbalizations were classified according to seven categories. Asking for clarification of rules and comments on the apparatus were the two categories with the highest frequencies.

Solution procedures that had been used in the measurement division tasks only in the cross-sectional sampling were evident in the longitudinal data for the partitive division tasks. These procedures involved a minimum or no manipulatory actions and were employed mostly by the older subjects. Two new procedure categories were thus added for the cargo partitive division task and three for the animal partitive division task in the longitudinal analysis. One new category had to be added for the animal measurement task.

DISCUSSION AND IMPLICATIONS OF RESULTS

Summary statement of the findings were made at the conclusion of Chapter III. A discussion on each of these follows.

<u>Children were more successful with the measurement division task associated with the animal groups task than with that associated with the cargo groups</u>.

When the cargo groups problem and the animal groups problem were designed it was intended that the mathematical structure of the measurement division tasks would be the same for both. In each, children were required to find how many groups of a given number could be made from a given set of objects. Differences in the physical structure of the situations, however, resulted in different behaviors on the part of the subjects at least up to age seven.

Most of the children in the age range four years to seven years were able to form the required groups in both tasks. Yet a greater proportion of the children in the age range correctly solved the animal groups measurement division problem. The cargo groups task was presented first in the cross-sectional sampling so one might suspect that the greater ease in solving the animal groups problem could be the result of learning in the first problem. However in the longitudinal sampling the animal groups measurement division problem was given first and it still proved to be easier.

The information given to the subjects in both tasks followed the same sequence. It was indicated first that all the objects (cars or animals) were involved in a move from one place to another. Then subjects were told that a given number of objects were to be grouped

together. Finally it was indicated that they would have to find the number of such groups.

In the animal groups task the animals were to be placed into cages. In order to answer the question once the task was completed the child could just count the cages that had been required. On the other hand in the cargo groups problem the cars were taken across on the ferry and placed in the second parking lot. Most subjects placed the cars in this parking lot in a random manner and the integrity of the groups was lost. When the child was required to think about the number of trips there were no groups to count. Subjects who had not thought to count the number of trips while manipulating the material were unable to give a solution.

Zweng (1963) made a study of these two types of measurement division problems and found no difference in difficulty for second grade children. The children found it as easy to do measurement division problems in which a group of objects was not associated with other objects as when these objects were associated with other objects, such as balloons in bags or animals in cages.

In applying the criteria for 'good' problems the cargo and animal groups measurement tasks were assumed to be equivalent as far as solving the problems was concerned. This may have been the case for some seven-year-olds and for most eight- and nine-year-olds because they did not appear to lose sight of the problem. However the difference in structure for the two tasks appears to have made a difference for the younger children. Many of these subjects appeared to focus on the manipulative aspect of the tasks and forget

the problem.

Piaget (1968) has pointed out that the pre-operational (ages 4 to 7) child's domain is still that of action and manipulation. His many studies also show how children at this age level focus, or centre, their attention on one aspect of the material and neglect other important aspects. Bruner et al (1966) also observed the one track tendency of the young child's mind.

Almy (1966) has observed that:

> . . . the child's responses are shaped not so much by the question put to him, as by his way of looking at the materials and objects in the experiment (p. 133).

She also found that questions that are apparently confusing to most kindergarten children are "crystal clear" to most second graders. Lovell (1975) acknowledges that the structure of a problem may affect its difficulty for children.

If children are to understand the process of the transformations that they bring to objects, it would seem reasonable to assume that they must reflect upon the results of their actions. It is only in this way that the young child may have logical-mathematical experiences as referred to by Piaget. Although the two measurement division problems in the present study were mathematically equivalent they did not provide for equivalent experiences. The perceptual aspect of the animal groups task aided the subjects to reflect upon the result of their interaction. They could look at the cages and see how many groups they had made. This was not the case with the cargo groups problem because the integrity of the groups was lost.

In devising equivalent problem-solving situations for young

children, care must be taken to monitor the kind of experiences that they will provide.

<u>The structure of the two animal groups tasks had many similarities. However the subjects had less success with the partitive division task.</u>

Because of the many similarities of the two animal groups tasks, these seem to be appropriate problems to compare the difficulty of the measurement and partitive division processes. The structure of the two cargo groups tasks may make these invalid problems for making such a comparison of the processes involved.

In the first task with the cargo groups the cars were not associated with another set of objects while they were in the second task. Both animal groups tasks on the other hand consisted of the same two sets of objects, the animals and the cages. The condition under which the mathematical processes were applied in the animal groups tasks were similar in many ways. The same apparatus was employed for both problems and the subjects were required to place animals into cages in both tasks.

The proportions of solution/no solution for the two animal groups tasks in the cross-sectional sampling were comparable. In the longitudinal sampling, however, where a greater number of subjects attempted the animal groups problem more children solved the measurement division task than the partitive division task. The eight- and nine-year-olds appeared to have no difficulty with either task, but several subjects up to age seven failed to solve the partitive division task after being successful with the first task with the animal groups.

Zweng (1963) found no significant difference in difficulty of problems involving measurement and partitive division for grade two children when objects of one set were to be associated with objects of another set. The present study revealed that most seven-year-olds and all eight- and nine-year-olds were equally successful with both animal groups tasks. Comparison of solution results for the two tasks for the subjects in the age range seven to nine does not dispute Zweng's findings. Solution results, however, should not be the sole criterion in comparing difficulty of the two processes involved.

Examination of the procedures employed by the subjects while attempting the various tasks revealed that three new categories had to be added for the animal partitive task in the longitudinal analysis. These procedures involved sophisticated strategies and were used only by the eight- and nine-year-olds in this task. Similar procedures, however, had previously been used by some younger subjects in the measurement division task. All three procedures had been employed with the cargo measurement division task and two had been used with the animal measurement division task in the cross-sectional sampling. They had not shown up in the cross-sectional data for the partitive division tasks. It appears that certain procedures used by the subjects to solve the measurement division problems were not available to them until they were eight or nine years of age when the mathematical process involved was partitive division. The findings seem to indicate that the partitive division process may be more difficult than the measurement division process for young children.

The various tasks contained distractions to which many of the subjects responded.

In designing the problem-solving situations used in the study an attempt was made to incorporate features that would make them interesting for the children. However, all problems could be solved with a minimum of manipulative actions. The data revealed that most subjects up to age seven interacted with the material to a greater extent than was required to solve the problems. It seems that many subjects were paying more attention to the irrelevant aspects of the tasks than to the given problems.

Gollin (1968) has pointed out how young children's behaviors are dominated by perceptual aspects of the materials. He states:

> In problem-solving and learning situations, subjects—particularly young subjects—are likely to attend to perceptually salient properties of the stimulus array. If the subject is asked to solve a problem or accomplish some goal with the stimulus materials, his initial manipulatory efforts are likely to be dominated by the perceptual properties of the materials (p. 61).

Analysis of the data for the problems under consideration tends to support Gollin's observations. It reveals, however, that all subjects do not respond to the perceptual properties in the same way. Boys had a greater tendency than the girls to simulate in minute detail real world situations in the cargo groups problems. Boys also appeared to be distracted more by the lion in the animal groups problems while the girls paid more attention to the smaller animals.

Although the data are meager, there are indications that some responses to distractions may be of an idiosyncratic nature.

Stevenson (1975) has pointed out the paradox we face in designing materials for young children. In an effort to make the problem interesting for children we often include irrelevant details. These details help to capture the child's interest but at the same

time they may distract him from the information that is central to the problem. We face the same paradox in attempting to design problem-solving situations for young children. If problems are to be meaningful mathematically, the child should be able to differentiate what is important in a situation from what is irrelevant. Dienes (1963) states:

> If a situation generates more noise than the learner is able to cut through, learning the relevant abstraction becomes impossible (p. 161).

In order to help the child focus on the important aspect of a problem, Stevenson (1975) suggests eliminating as much irrelevant information as possible because young children have a difficult time doing this by themselves. Skemp (1971) on the other hand considers some noise as necessary to concept formation. He advocates little distracting details and clear embodiment of a concept in the early stages and then increasing the noise as the concept becomes more strongly established (p. 33). Dienes (1963) indicates that noise can be cut down by imposing restrictions on the child's freedom of action but that this may also eliminate some of the fun in solving problems (p. 55).

The problem appears to be one of maintaining interest while at the same time eliminating distractions beyond a child's capabilities. Dienes (1963, p. 161) defines the problem as that of finding the laws of interaction between three variables: (1) amount of noise generated, (2) amount of noise learner is able to cut through, (3) extent of motivation of the learner.

Dienes appears to have identified the major variables in the

creation of problem-solving situations for young children. Unfortunately, research data is not yet available to make definitive statements about the interactions and relationships among these variables.

The present investigation was not set up to differentiate those distractions that would prevent the children from solving the problems from those that would not. There are indications, however, that some irrelevant aspects of the tasks produced more noise than others. The color of the cars and of the houses, for example, appeared to be a very distracting element for some subjects. The lion was another source of distraction that some were unable to overcome. Other responses to the distractions, such as standing or classifying the animals, did not appear to prevent many subjects from solving the problems.

The findings of the present investigation suggest that the role of noise in problem-solving situations should be monitored very closely.

<u>The majority of the subjects used a systematic procedure in their attempt to solve the partitive divisions problems. Some procedures, however, may have been influenced by the requirements of the first task.</u>

More than half of the subjects used a systematic procedure in distributing the objects while attempting the partitive division tasks. Some subjects distributed the objects one at a time and others distributed them several at a time. The logical procedure for assuring that equal groups be made would seem to be distributing objects one by one. However, only about 12% of the subjects who distributed the objects into groups used this procedure. Although a

few five- and six-year-olds made a one by one distribution this behavior was exhibited mostly by the older subjects.

Zlot (1976) suggests introducing partitive division before measurement division "because most children can more quickly grasp the reality" of these types of situations (p. 124). Most young children have had the experience of sharing objects equally among friends either in games or in play. However, the rules of partition are often imposed upon them without requiring them to reflect on what the process accomplishes. Dividing objects into equal groups may not be a natural process for children and their experiences may not transfer to problem-solving situations. The present investigation suggests that young children do not have a tendency to distribute objects one by one.

Other systematic procedures permitted many subjects to solve the partitive division problems correctly. Some of the procedures employed, however, suggest that the requirements of the first task may have influenced several subjects' choice of procedure in attempting the second task. Some subjects started the task by placing three cars at each house or five animals in each cage. Such behavior denotes the influence of one task over another.

It could be hypothesized that some children knew that they could make groups of three or five objects in the second task of a problem because they had done so in the first task. This appears to indicate that subjects who related tasks in this manner were focusing on some aspect of the groups with which they were familiar. It would seem reasonable to assume that such interaction with the material may permit simple abstraction to take place but may not be the basis for reflective abstraction.

The present investigation suggests that we need to know more about how one task may influence strategies employed in another task. Such a dependance on another task for choice of procedure may be desirable in certain tasks but undesirable in other problem-solving situations. It would be interesting also to know how children would behave if the number of objects did not permit solution of a problem by using such procedures in division problems.

> <u>The younger subjects had a tendency to state that the groups that they had made were equal when they were not or when the partitioning had been done correctly they tended to count the objects in each group even after stating that each contained the same number</u>.

The requirements of the partitive division tasks were that equal groups be made. Several subjects, up to age six, did not partition the objects so that there were the same number in each group. Yet when asked if the groups contained the same number they indicated that they did. Some subjects, including six-year-olds, still maintained this to be true even after they counted the objects in each group.

At times the number of objects in each group only varied by one or two. At other times, however, when the number varied considerably, some subjects still maintained that the groups were equal.

Piaget, in his many studies, has shown how a child at the preoperational stage is swayed by perceptual cues and is often unaware when his conclusions are self-contradictory. Perceptions may account for some subjects' incorrect responses in the partitive division tasks when the number of objects varied only slightly in each group. It does not seem to account for their responses, however, when there were many more objects in one group than there were in the others. One might assume that some children wanted to maintain the rules imposed by the interviewer in spite of the fact that their manipulatory actions had

not permitted them to do so. Behaviors that might be related to these were exhibited by a few six-year-olds in the cargo measurement division task. Some subjects in this age group maintained that the ferry had made the number of trips that they had predicted even if it was not correct.

When equal groups were made and the subjects indicated that they were equal, they were asked how many objects each group contained Almost all subjects up to age six inclusive counted the objects in each group before stating the answer. It is possible that these subjects were not convinced that the groups were equal and they had to count the objects in each. It would seem equally reasonable to assume that these subjects were able to make gross quantitative comparisons but were unable to see the numerical relationship between the sets. Almost all seven-, eight- and nine-year-olds counted the objects in one group only and gave their answer. They appeared to know that each group contained the same number.

The findings seem to indicate that there is a marked difference in children's understanding of numbers and their relationships when they are six years of age or younger and when they are seven. Piaget has pointed out that children do not understand numbers until they are able to conserve quantity which usually occurs around age six or seven. Meager as the evidence may be, the results of this investigation tend to confirm that children at the pre-operational stage do not have a command of numbers. This seems to imply that when young children are given mathematical problems insistence on correct numerical solutaions may be meaningless to them.

<u>The children's procedures changed and became more sophisticated as they got older.</u>

Categories were established for the procedures employed by the subjects in each task. They were generally ordered in a hierarchy from what was judged to be the lowest to the highest level of solution procedure used. (There were notable exceptions to this hierarchy which have already been pointed out.)

Most three-year-olds were unable to follow instructions. Some subjects did manipulate the objects but their actions resembled play in that they did not appear to be seeking a solution to the problem.

If mathematics instruction is to start at an early age it would seem appropriate to accept Dienes' first intructional stage in concept formation. The age level at which the play stage is advocated could possibly vary for different concepts.

The four- and five-year-olds had more control over their manipulations but they had a tendency to simulate actions with which they were familiar. All subjects in this age range attempted to solve the problems by going through the motions of taking cars on the ferry, placing them at the houses and of putting animals into cages.

The six-year-olds used a wide range of procedures. Although many subjects in this age range still used procedures that were employed by the four- and five-year-olds, a few subjects solved the problems by making groups on the table and some even solved problems without any manipulation of the objects. One by one distribution of the objects, however, was only used in a few instances by subjects who were age six. It should be recalled that the six-year-olds of the cross-sectional group were probably not representative of this age group.

Some seven-year-olds went through the actions of displacing objects from one place to another but a greater number solved the problems by making groups on the table or without any manipulation. Most eight-year-olds displayed sophisticated procedures in solving the problems. Many subjects in this age group either made groups on the table or gave the solution without manipulation. All nine-year-olds used the higher categories of solution and most solved the problem without manipulating the objects.

The findings seem to indicate that children start using sophisticated procedures around age six. These procedures, however, appear to be available only to a few at this age level. The number of subjects using such procedures increased at ages seven to nine.

<u>The children displayed a variety of verbalization behaviors while attempting the problems</u>.

It was not the primary purpose of this study to investigate verbalizations. The data revealed, however, that spontaneous verbalizations are part of the behaviors of children as they go about solving problems.

Not all subjects exhibited this behavior and those who did did not do so in every task. Klein (1964) studied children's spontaneous verbalizations as they worked alone. He concluded that:

> . . . whether or not a child talks to himself when alone may be related to a complex interaction of developmental readiness, personality and task variables (p. 50).

The problem-solving situation in the present study involved interaction with an interviewer. In Klein's investigation the subjects worked behind a one-way mirror. There is no reason to believe, however, that the interactions among the three variables identified by Klein would be markedly different in the study reported here.

[illegible]

We may have no control over which aspects of a task will elicit children's verbalizations, but a close monitoring of such behaviors may reveal on which aspect they focus. Many studies have been undertaken where children were required to think aloud as they attempted problems. Donaldson (1963) has pointed out pitfalls of such investigations. Commenting on the thinking aloud process, she states:

> . . . what if verbalizing actually changes the thoughts? We have to admit it may be true that when a subject thinks aloud he will think in a way that is different from that in which he otherwise would have thought (p. 30).

Spontaneous verbalizations may be a more appropriate means of learning how children approach problems.

The findings of the present investigation on children's spontaneous verbalizations should be studied further. However, some of the subjects' verbalizations seem to indicate aspects of the problems that were important to them. The many questions asked by the older subjects appear to indicate a concern for clear understanding of the requirements of a task or of a problem. The results also showed that many questions appeared to have been prompted by the requirements of the task that the subjects had just completed. There is a possibility that using the same apparatus for two or more different tasks may tend to confuse children.

Comments on the apparatus support the findings that young children have a tendency to get involved in the irrelevant aspects of a task. As has been observed in many studies, the younger children had a tendency to monitor their actions. Piaget (1968) has observed how the small child constantly talks to himself and he hypothesizes that some monologues "serve as an adjunct to immediate actions (p. 21)."

Verbalizations classified as rule making were exhibited mostly by the younger subjects. This behavior appears to support the contention that young children have a tendency to interpret a task as they perceive it. It also seems to indicate that they often impose their own rules upon certain irrelevant aspects of a task and that this may distract them from the problem.

The findings of the present investigation would seem to imply that protocols in problem-solving situations should be made as simple and as clear as possible so that children understand what is required of them. There are also indications that tasks which are appropriate for four- and five-year-olds may not be suitable for three-year-olds. Devising appropriate tasks for children at age three may be difficult and needs further investigation. The findings also seem to indicate that much can be learned by children's spontaneous verbalizations while they attempt to solve problems. Close monitoring of such behaviors may permit further refinements of criteria for problem-solving situations.

CONCLUDING STATEMENTS

The findings of the present study are not conclusive but they do reveal many things that young children do as they go about solving non-verbal problems. The following general concluding statements seem to be supported by the data. These statements may need to be checked by further research.

1. The physical structure of mathematically equivalent problems designed for young children can make some more difficult to solve than others.

2. It appears that the partitive division process requires a higher level of mental operation than does the measurement division process.

3. In comparing difficulty of mathematical processes the problems should be made as similar as possible and solution results should not be the sole criterion.

4. The requirements of one task can influence young children's choice of procedure in attempting another task when the same apparatus is involved.

5. Mathematically irrelevant aspects of a problem may distract different children in different ways. Some responses to the distractions appear to be related to age and others to sex.

6. One by one distribution or partition of objects does not appear to be a procedure that children employ in division problems before they are six or seven year of age.

7. There are indications that most children up to age six who are able to establish the relation of set equivalence through perception or manipulation cannot transfer the information to the cardinality of the sets.

8. Spontaneous verbalizations expressed by children while solving problems appear to be elicited mostly by children while they are young and seem to be more task related when they are older.

9. The procedures that children employ in attempting to solve division problems change from a highly manipulative aspect to a cognitive aspect in the four to nine age range.

RECOMMENDATIONS FOR FURTHER RESEARCH

The present investigation has prompted many questions. The area most in need of answers at the present time appears to be the role that distractions play in problem-solving situations.

Researchers should attempt to determine what aspects of problems distract young children. Are the distractions of an idiosyncratic or of a nomothetic nature? Should problems be sequenced according to increasing noise levels or should children be given the opportunity to cut through the distractions when a concept is first introduced? Are the effects of either approach the same for all concepts? What effect does decreasing noise have on maintaining interest? Answers to these questions must be found.

The two problems used in the investigation consisted of a measurement and a partitive division task. The same apparatus was used in both tasks of one problem. It was shown that the requirement of the first task of a problem often appeared to influence the subjects' choice of procedure in attempting to solve the second task. Further studies should be undertaken to see if this is an advantage or a disadvantage in learning situations. Other studies may attempt to determine whether this behavior only occurs when the same material is used.

The data appeared to show that the partitive division process was more difficult that the measurement division process for young children. It is not known, however, whether it would be advantageous to expose children to the processes separately or simultaneously. Only longitudinal studies can furnish an answer to this question.

It was concluded that many of the younger subjects had

difficulty solving the cargo measurement division task because the integrity of the groups was lost. Children usually experience situations similar to these in the real world. Further research should attempt to determine whether such problems should or should not be included in a mathematics program for early childhood. Would experience with this type of problem help children think about the mathematical ideas involved? This question deserves investigation.

A laboratory set-up was used in this study to interview the subjects. Attempts may be made in further research to determine if similar results would be obtained using a looser protocol and a different environment.

The problems used in the Nelson and Sawada (1974) investigation were designed according to certain criteria for "good" problems in early childhood education. The findings of the present study suggest that some of these criteria require certain refinements and that other changes may need to be made after further research is undertaken.

One of the criteria established by Nelson and Kirkpatrick (1975) was that the situation in which the problem occurs should involve real objects or obvious simulations of real objects. However, the present study has shown that such objects may be a source of noise for children. Further research into the precise nature of noise in problem-solving situations may necessitate changes in this criterion. For the present it can be noted that several of the older subjects were able to solve the problems mentally. This would seem to indicate that this criterion may be more relevant for children up to age six or

seven than for eight- and nine-year-olds.

Closely related to the criterion on real objects is the one which stipulates that children themselves should manipulate the material. The subjects interviewed in the present study were free to manipulate the objects. The findings revealed that several subjects solved the measurement division tasks without manipulating the material or with a minimum of interaction but were unable to solve the partitive division tasks in this manner. This would seem to indicate that the importance of manipulation on the part of the children, besides being related to age, may also be related to the concepts involved. The criterion could be changed to read: The problem should involve a transformation and the child should be free to manipulate the material if he wishes.

Another criterion provides for a problem (i.e. mathematical concept) to have many physical embodiments. The findings for the two measurement division tasks have shown that the embodiments can change the type of interaction that takes place. Some experiences appear to provide for reflective abstractions more than others. Although no changes for this criterion are suggested by the data, it would appear that problem situations with different embodiments need to be designed carefully.

In conclusion, it is important to note that the other four criteria appear to have been met by the problem situations designed for this study.

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APPENDICES

APPENDIX A

PROTOCOLS

1. Cargo Groups - Cross-Sectional Sampling
2. Animal Groups - Cross-Sectional Sampling
3. Cargo Groups - Longitudinal Sampling
4. Animal Groups - Longitudinal Sampling

CROSS-SECTIONAL SAMPLING

CARGO GROUPS

The apparatus is placed on a table in front of the child who is standing or seated. The model is referred to and the child is shown the river, the islands, the ferry boat, the parking lots and the houses. There are fifteen plastic cars in the parking lot on one side of the river and three houses placed on the other side of the river. The child is first shown how the ferry can cross the river and is asked to choose a car, put it on the ferry, take it off the ferry, and park it in the parking lot on that side. Assistance is given the child with these moves if necessary. When they are completed the car is returned to the first parking lot.

The child is then told that all the cars are to be taken across the river and parked in the second parking lot. He is advised that the ferry accommodates exactly three cars each trip and the following question is asked: "If the ferry boat can take only three cars each trip, how many trips must the ferry take to get all the cars across?" No assistance is given to the child except to remind him of the original question if he puts too many or two few cars on the ferry boat. When the child indicates he has finished the operation he is asked: "How many trips did the ferry boat take?" If all the cars are not on the second parking lot they are now assembled there.

The child is asked then to park all the cars beside the three houses so that there are the same number of cars at each house.

If the partitioning operation offers some difficulty he is reminded of the original problem. When the child has parked all the cars, he is asked: "Does each house have the same number of cars? How many cars at each house?"

CROSS-SECTIONAL SAMPLING

ANIMAL GROUPS

The board is placed on a low table in front of the child so that he has an overview of it. A box containing 26 posts and 17 slats for fence building, and another box containing an assortment of 20 toy animals are placed between the child and the board. The child is shown how the posts fit into the holes, which were designed to accommodate them, on the board. He is also shown how a slat fits between two posts to make a fence. The child is asked to build two more fences, like the one he was shown, anywhere on the board. When the child has completed these, he is asked to build a closed corral (cage, pen) to keep some animals in. If he has any difficulty constructing this cage, he is given assistance.

The child is then presented with the box of animals which includes four camels, four ducks, four mice, four hippopotamuses, an elephant, moose, horse and a lion. He is instructed to build enough cages for these animals with the specific directions that all the animals must be put in the cages, and that there are five animals in each cage. The child is asked, "If there are five animals in each cage, how many cages will you need for all these animals?" Only if the child encounters difficulty with the problem or appears to have forgotten the problem, he is reminded of the original question. Otherwise no comments are made about his performance. When he indicates that he has finished the task, the child is asked "Are there the same number in each cage?" "How many cages are there?"

The animals are collected, the two camels are removed, and the remainder (18) are placed in the box. One of the cages is dismantled so only three cages remain. The child is told that the remaining cages are for the animals in the box. He is asked to put all the animals in the cages so that there are the same number of animals in each cage. If the child has difficulty with this operation or appears to have forgotten the problem, he is reminded of the original question. When the child appears to have completed the task, to his satisfaction, he is asked "Are there the same number in each cage?" "How many animals are there in each cage?"

LONGITUDINAL SAMPLING

CARGO GROUPS

The apparatus is placed on a table in front of the child who is standing or seated. The model is referred to and the child is shown the river, the islands, the ferry boat, the parking lots and the houses. There are fifteen plastic cars in the parking lot on one side of the river and three houses placed on the other side of the river. The child is first shown how the ferry can cross the river and is asked to choose a car, put it on the ferry, take it off the ferry, and park it in the parking lot on that side. Assistance is given the child with these moves if necessary. When they are completed the car is returned to the first parking lot.

The child is then told that all the cars are to be taken across the river and parked in the second parking lot. He is advised that the ferry accommodates exactly three cars each trip and the following question is asked: "If the ferry boat can take only three cars each trip, how many trips must the ferry take to get all the cars across?" If the child does not appear to be interacting at any level then after a period of about fifteen seconds ask: "Would you like me to repeat the question?" and repeat as necessary. If the child still appears to be stymied and unable to give a response suggest that he can load the cars on the ferry and take them across if he wants to. If the child loads other than three cars on the ferry ask: "How many cars were you supposed to put on the ferry?" and give the answer as necessary. Provide this information only

once. Further help is restricted to a reminder of the original question. When the child indicates he has finished the operation he is asked: "How many trips did the ferry boat take?" If all the cars are not on the second parking lot they are now assembled there.

The child is asked then to park all the cars beside the three houses so that there are the same number of cars at each house. If the partitioning operation offers some difficulty he is reminded of the original problem. When the child has parked all the cars, he is asked: "Does each house have the same number of cars? How many cars at each house?" Be sure to ask these questions in the same order for all children.

LONGITUDINAL SAMPLING

ANIMAL GROUPS

The board is placed on a low table in front of the child so that he has an overview of it. A box containing 26 posts and 17 slats for fence building, and another box containing an assortment of 20 toy animals are placed between the child and the board. The child is shown how the posts fit into the holes, which were designed to accommodate them, on the board. He is also shown how a slat fits between two posts to make a fence. The child is asked to build two more fences, like the one he was shown, anywhere on the board. When the child has completed these, he is asked to build a closed corral (cage, pen) to keep some animals in. If he has any difficulty constructing this cage, he is given assistance.

The child is then presented with the box of animals which include four camels, four ducks, four mice, four hippopotamuses, an elephant, moose, horse and a lion. He is told that we would like to build enough cages to hold the animals. The child is asked: "If each cage holds five animals how many cages will we need?" If the child does not appear to be interacting at any level then after an interval of approximately fifteen seconds ask, "Would you like me to repeat the question?" and repeat as necessary. If the child still appears to be stymied and unable to make a response suggest that he build the cages and put the animals in them. If the child puts other than five animals in each cage ask, "How many animals were supposed to go in each cage?" and give the answer as necessary.

Provide this information only once. Further help is limited to a reminder of the original question. When he indicates that he has finished the task, the child is asked "Are there the same number in each cage?" "How many cages are there?"

The animals are collected, the two camels are removed, and the remainder (18) are placed in the box. One of the cages is dismantled so only three cages remain. The child is told that the remaining cages are for the animals in the box. He is asked to put all the animals in the cages so that there are the same number of animals in each cage. If the child has difficulty with this operation or appears to have forgotten the problem, he is reminded of the original question. When the child appears to have completed the task, to his satisfaction, he is asked "Are there the same number in each cage?" "How many animals are there in each cage?" The order in asking these questions should be the same for all children.

APPENDIX B

CODE USED FOR RECORDING DATA FROM VIDEOTAPE

CODE FOR RECORDING DATA ON CARGO GROUPS PROBLEM

E	Protocol question by E (examiner).
E_1	Question and gesture.
E_2	Question and demonstration.
E_3	E asks subject to make trial run.
E_4	Question by E during task.
E_5	Praise by E.
E_6	"How many trips did the ferry boat take?"/ "Does each house have the same number of cars?"
E_7	"How many cars at each house?"
E_F	E parks ferry.
E_B	E places car on ferry.
E_C	E removes car from ferry.
S_T	S (subject) makes trial run.
V_1	S asks for clarification.
V_2	S repeats question to self.
V_3	Comment pertaining to material/task.
V_4	S makes motor sound.
V_5	Comment not pertaining to task.
Y	Nod for yes.
V_Y	Nod and verbal yes.
N	Nod for no.
V_N	Nod and verbal no.
R	Right hand.
L	Left hand.
A	S pushes car.

B	S lifts car.
T	Touching.
P_L	Pause, looks.
↑	S picks up a car and holds.
↶	S picks up a car and places back on table.
F_1	Ferry placed perpendicular to shore.
F_2	Ferry placed parallel to shore.
F_3	Ferry not touching/corner touching shore.
F_4	Docks ferry at previously docked place.
F_5	Docks ferry at a spot opposite unoccupied space in parking lot.
K	Places car in unoccupied spot.
M	Pushes previously parked cars to make room for arriving cars.
F_*	Brings ferry onto shore, lifts back, cars slide off.
X	S turns car/ferry around.
C_1	S crosses ferry directly.
C_2	S pushes ferry around island.
⊗	S chooses a car.
→	S repeats previous action.

CODE FOR RECORDING DATA ON ANIMAL GROUPS PROBLEM

E_{D1}	E demonstrates how posts fit into holes.
E_{D2}	E demonstrates how slats fit between posts.
E_F	E asks S to build two more fences.
E_{F1}	E helps S placing a post.
E_{F2}	E helps S placing a slat.
E_A	E asks S to build a cage—no demonstration.
E_B	E asks S to build a cage—demonstration: E_{D1} or E_{D2}.
S_F	S builds one or two fences.
S_B	S builds a cage.
E_H	E hands animal to S.
E_1	"If there are five animals in each cage, how many cages will you need for all these animals?"/"Put all the animals in the cages so that there are the same number of animals in each cage."
E_{1G}	"If each cage holds five animals how many cages will you need?"
E_3	Prompting by E.
E_4	Question by E during task.
E_5	Praise by E.
E_6	"Are there the same number in each cage?"
E_7	"How many cages are there?"/"How many animals are there in each cage?"
V_1	S asks for clarification.
V_2	S repeats question to self.
V_3	Comment pertaining to material/task.
V_5	Comment not pertaining to task.
Y	Nod for yes.
V_Y	Nod and verbal yes.

N	Nod for no.
V_N	Nod and verbal no.
R	Right hand.
L	Left hand.
M	Move/picks up.
T	Touching
P_L	Pause, looks.
↑	S picks up an animal and holds.
↶	S picks up an animal and places back on table.
*	S stands animal inside cage.
⊗	S chooses an animal.
[⊗]	S chooses same kind of animal as previous one.
Θ	S stands an animal that has fallen inside a cage.
O	S moves an animal that is standing inside a cage.
→	S repeats previous action.

APPENDIX C

SAMPLE OF RECORD SHEETS

1. Cargo Groups - Measurement Division
2. Cargo Groups - Partitive Division
3. Animal Groups - Measurement Division
4. Animal Groups - Partitive Division

Cargo Groups - Measurement Division

Name *P* C.A. *6-0* Task *1*

Cross sectional

E demonstrates — crosses one car, E asks S to bring it back, he does. E V_3 ("Why?") E ("... would sink..") E_1 P_L V_3 ("em") V_3 ("20") E ("you think 20, you try, you take all your cars across")

	F	LOADING CARS	CROSS. FERRY	F	UNLOADING CARS	CROSS. FERRY
1	F_1	[$1^1\ 1^3$ / —2] L_{A_1} L_{B_1} →	C_1	F_1	[$1^1\ 1^3$ / —2] L_{A_1} (X) L_{B_3} L_{A_2} P(sits) E ("want to take these across too" — E points to c. in 1st p.l.)	C_1 (over the island) X
2	F_3	[1_1 / $1_2\ 1_3$] L_{B_1} L_{B_2} (#2+3)	C_1	F_{14K}	[1_1 / $1_2\ 1_3$] L ↑2 (#2+3) L_{B_2} L_{B_3} L_{B_1}	C_1
3	F_1	[1^3 / $1^1\ 1^2$] R_{B_1} → →	C_1 X (no fit)	F_{14K}	[$1^2\ 1^1$ / 1^3] L_{B_1} L_{B_2} L_{B_3} P_L (around room) (Short)	C_1
4	F_3	[$1_1\ 1_3$ / 1_2] L_{B_1} → (L→R_{B_1})	C_1	F_{34K}	[$1_1\ 1_3$ / 1_2] (L→R_{B_1}) (L→R_{B_3}) X (L→R_{B_2})	C (lifts over island)
5	F_1	[1^1 / $1^2\ 1^3$] L_{B_1} → →	C_1 X (no fit)	F_{34K}	[$1^3\ 1^2$ / 1_1] L_{B_2} L_{B_3} L_{B_1}	

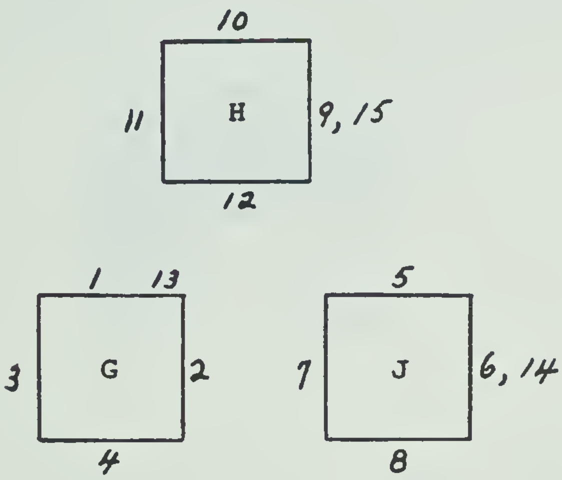
Cars were all lined up in a row in 1st p.l. to begin task (E + S had arranged them thus) S arranged cars in 2nd p.l.

E_5 E_6 V_3 ("twenty")

Cargo Groups - Partitive Division

Name R. C.A. 5-6 Task 5

Longitudinal

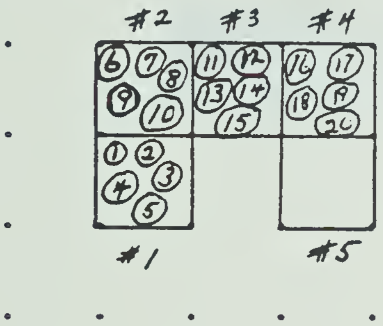


E_1 V_3 ("I don't know") R_{A_1} V_3 ("one") R_{A_1} $\rightarrow$ R_{B_1} R_{A_1} $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ R_{B_1} $\rightarrow$ $\rightarrow$ E_6 V_3 ("yeah") E_7 R_T (each car at J) V_3 ("one, two, three, four, five") R_T (each car at G) V_3 ("one, two, three, four, five") R_T (each car at H) V_3 ("one, two, three, four, five") E_5

Animal Groups - Measurement Division

Name W. Longitudinal C.A. 8 - 5 Task 3

$E_{D\,1+2}$ E_F S_B (#1) E_{1G} $\underset{(8\ \text{secs.})}{P}$ V_3 ("five") E ("5? ok lets try that") E ("you start building your cages") S_B (#2, 3, 4) E_4 ("want some more cages?") Y S_B (#5) E ("ok, remember I want 5 ani. in each cage")

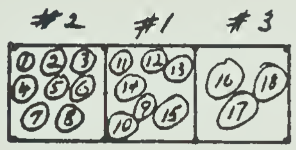


$R_{m_1}^{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ Θ $R_{m_1}^{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$ E_4 ("ok, we got five in each cage?") V_3 ("yeah") E_7 ("how many cages did we need?") V_3 ("four")

Animal Groups - Partitive Division

Name J. Cross sectional   C.A. 4-2   Task 6

E P Y E_1 $\underset{\curvearrowleft_1}{L}$ $\underset{\uparrow_1}{R}$ V_3 ("I gonna ... this horsey ...")



$L^*_{m_1}$ $\xrightarrow{*}$ $\underset{[\bullet]}{\xrightarrow{*}}$ $\xrightarrow{*}$ $\underset{\odot}{\xrightarrow{*}}$ (Same as #3) $\xrightarrow{*}$ $\xrightarrow{*}$ R_{m_1} (#8) $\xrightarrow{*}$ $\xrightarrow{*}$ $\xrightarrow{*}$

$L^*_{m_1}$ $R^*_{m_1}$ $L^*_{m_1}$ $\xrightarrow{*}$ $R^*_{m_1}$ $\xrightarrow{*}$ $\xrightarrow{*}$ R_p (cage #3)

V_3 ("... are three") E ("we got all the animals in cages") Y E_6 Y R_p (cage #3) V_3 ("these are three") E ("three there") R_p (cages #1 + 2) V_3 ("five") E ("and I asked you to put the same no. in each cage") Y E_4 ("is that what you got?") Y E_7 R_p (each animal in cages #3, 1, 2)

V_3 ("1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18") E_5.

APPENDIX D

ALLOCATION OF PROBLEM SETS TO SUBJECTS FOR EACH AGE GROUP

ALLOCATION OF PROBLEM SETS TO SUBJECTS FOR EACH AGE GROUP:

CROSS-SECTIONAL SAMPLING

Problem Set	Problems					
# 1	1	7	5	3	2	4
# 2	5	3	1	9	6	2
# 3	3	11	1	5	12	2
# 4	3	9	7	1	10	2
# 5	1	7	3	11	2	8
# 6	9	1	3	11	4	12
# 7	7	9	1	5	8	6
# 8	11	7	5	1	12	6
# 9	1	5	11	9	6	10
#10	9	1	11	7	12	8
#11	5	3	9	7	4	8
#12	11	5	7	3	6	4
#13	9	3	5	11	10	4
#14	7	11	9	3	8	10
#15	5	11	9	7	12	10

*1. Cargo groups
*2. Animal groups
3. Parking lot
4. Theatre grid
5. Linear sequence
6. Circular sequence
7. Object reflection
8. Mirror reflection
9. Factor platform
10. Factor board
11. Fold-up shapes
12. Projected shapes

*Problems in this study.

ALLOCATION OF PROBLEM SETS TO SUBJECTS FOR EACH AGE GROUP:

LONGITUDINAL SAMPLING

Problem Set	Problems					
# 1	2	8	6	4	1	3
# 2	6	4	2	10	5	1
# 3	4	12	2	6	11	1
# 4	4	10	8	2	9	1
# 5	2	8	4	12	1	7
# 6	10	2	4	12	3	11
# 7	8	10	2	6	7	5
# 8	12	8	6	2	11	5
# 9	2	6	12	10	5	9
#10	10	2	12	8	11	7
#11	6	4	10	8	3	7
#12	12	6	8	4	5	3
#13	10	4	6	12	9	3
#14	8	12	10	4	7	9
#15	6	12	10	8	11	9

*1. Cargo groups
*2. Animal groups
3. Parking lot
4. Theatre grid
5. Linear sequence
6. Circular sequence
7. Object reflection
8. Mirror reflection
9. Factor platform
10. Factor board
11. Fold-up shapes
12. Projected shapes

*Problems in this study.

PROBLEMS ATTEMPTED BY EACH SUBJECT IN THE CROSS-SECTIONAL AND LONGITUDINAL SAMPLINGS

Age	Subj.	1	2	3	4	Age	Subj.	1	2	3	4	Age	Subj.	1	2	3	4
	1	x	x				1	x	x	x	x		1	x			x
	2	x	x				2	x			x		2	x			x
	3	x	x	x	x		3	x			x		3	x	x		
	4	x			x		4	x					4	x			x
3-4	5	x				4-5	5	x	x			5-6	5	x	x	x	x
	6	x					6	x	x	x	x		6	x			x
	7	x			x		7	x					7	x			x
	8	x	x	x	x		8	x					8	x	x	x	x
	9	x			x		9	x	x	x	x		9	x	x	x	x
	10	x	x				10	x	x				10	x	x	x	x
Total		10	5	2	5			10	5	3	5			10	5	4	9

Age	Subj.	1	2	3	4	Age	Subj.	1	2	3	4	Age	Subj.	1	2	3	4
	1	x			x		1	x			x		1	x	x		
	2	x	x	x	x		2	x	x	x	x		2	x	x	x	x
	3	x	x	x	x		3	x	x	x	x		3	x			x
	4	x			x		4	x			x		4	x			
6-7	5	x			x	7-8	5	x	x	x	x	8-9	5	x			x
	6	x	x	x	x		6	x			x		6	x	x	x	x
	7	x			x		7	x	x	x	x		7	x	x	x	x
	8	x	x	x	x		8	x			x		8	x			
	9	x	x	x	x		9	x			x		9	x			x
	10	x			x		10	x	x				10	x	x		
Total		10	5	5	10			10	5	4	9			10	5	3	6

1. Cargo groups problem - cross sectional
2. Animal groups problem - cross sectional
3. Cargo groups problem - longitudinal
4. Animal groups problem - longitudinal

APPENDIX E

TABLES

Table E-1

Distribution of Solutions for Subjects who Attempted the Four Task in the Cross-sectional Sampling
(N = 30)

		Cargo Groups		Animal Groups	
Age	Subject	Measurement	Partitive	Measurement	Partitive
	S_1				
	S_2				
3	S_3				
	S_4				
	S_5				
	S_1				
	S_2				
4	S_3			*	*
	S_4			*	*
	S_5		*	*	*
	S_1		*		*
	S_2			*	
5	S_3				
	S_4			*	*
	S_5		*	*	*
	S_1				*
	S_2				
6	S_3		*		
	S_4	*	*	*	*
	S_5	*			
	S_1	*	*	*	*
	S_2	*	*	*	*
7	S_3	*	*	*	*
	S_4		*	*	*
	S_5	*	*	*	*
	S_1	*	*	*	*
	S_2	*	*	*	*
8	S_3	*	*	*	*
	S_4	*	*	*	*
	S_5	*	*	*	*
Total	30	11	15	17	18

* - Solution

Table E-2

Distribution of Response to Distractions:
Cargo Measurement Division—Cross-
Sectional Sampling
(N = 60)

Age	Sex	Behavior Categories	Solution
3	M	2, 4	
3	M	4	
3	M	3	
3	M	4	
3	F	2, 3, 5	
4	M	1, 2, 6	
4	M	4, 6	
4	M	5, 6	
4	M	1, 3	
4	M	2, 5	
4	F	5	
5	M	2, 4	
5	M	3, 4, 5	
5	F	5	*
5	F	2, 3	
5	F	2, 3	
6	M	2, 5	
6	M	1, 2, 3	
6	M	1, 3	*
7	M	3	
8	M	3	*
8	F	3	*

1. Driving cars onto ferry
2. Turning ferry around
3. Docking ferry perpendicular to shore and driving cars off
4. Making motor sounds
5. Parking cars side by side
6. Pushing ferry around island

Table E-3

Distribution of Response to Distractions:
Cargo Partitive Division—Cross-
Sectional Sampling
(N = 60)

Age	Sex	Behavior Categories	Solution
3	M	1, 2, 3	
3	M	3	
4	M	1	
4	M	1	
4	M	1	
4	F	1	*
5	M	1, 2	*
5	F	1	
5	F	1	
6	M	1	
6	M	2	
6	M	1, 3	
6	M	1	*
8	F	1	*

1. Pushing cars
2. Parking cars on the road
3. Making motor sound

Table E-4

Distribution of Response to Distractions:
Animal Measurement Division—Cross-
Sectional Sampling
(N = 30)

Age	Sex	Behavior Categories	Solution
3	M	1, 3	
3	F	1	
4	M	3	
4	M	1	
4	M	1	*
4	F	1	*
4	F	1, 2	*
5	M	1	
5	F	1, 2	*
5	F	1, 2, 3	
5	F	1	*
5	F	1	*
6	M	1, 3	
6	M	1, 2, 3	
6	M	1, 2	
6	F	2	*
6	F	1	
7	M	1, 2, 3	*
7	M	1, 2	*
7	M	1, 3	*
7	F	1, 2	*
8	M	1, 2	*
8	M	1, 2	*
8	F	1, 2	*

1. Standing the animals
2. Classifying
3. Commenting on the animals

Table E-5

Distribution of Response to Distractions:
Animal Partitive Division—Cross-
Sectional Sampling
(N = 30)

Age	Sex	Behavior Categories	Solution
4	M	1	
4	M	1	
4	F	1	*
5	M	1, 3	*
5	F	1	
5	F	1	
5	F	1	*
5	F	1	*
6	M	1	*
6	M	2	
6	M	1, 2, 3	
6	F	1, 3	
7	M	1	*
7	F	1, 2	*
8	M	1, 2	*
8	M	1	*
8	F	1, 2	*
8	F	1, 2	*

1. Standing the animals
2. Classifying
3. Commenting on the animals

Table E-6

Distribution of Procedures Employed by Subjects Attempting Cargo and Animal Groups Problems in Both Samplings (N = 21)

		Cargo Groups				Animal Groups			
		Cross-sectional		Longitudinal		Cross-sectional		Longitudinal	
Age	Subject	M	P	M	P	M	P	M	P
3-4	S_1	II	I	II	I	IV	I	I	I
	S_2	IV	IV	IV	III	II	I	III	VIII
4-5	S_1	IV	IV	IV	III	III	II	II	VII
	S_2	IV	IV	IV	VI	V	V	V	VI
	S_3	IV	IV	II	VI	VII	V	V	I
5-6	S_1	IV	IV	VIII	VI	VII	IV	V	VIII
	S_2	IV	IV	IV	V	V	VIII	IX	VI
	S_3	III	V	V	V	V	V	V	VI
	S_4	III	IV	VII	VI	III	IV	VI	VI
6-7	S_1	IV	II	V	VIII	III	VI	V	II
	S_2	IV	III	III	--	II	III	V	VII
	S_3	III	VII	IV	III	IV	III	V	IV
	S_4	VII	VII	VII	IX	V	VI	IX	VIII
	S_5	V	IV	V	III	IV	II	VI	VI
7-8	S_1	VIII	VIII	VII	IX	VI	VI	VI	VII
	S_2	VII	VI	VIII	VI	VIII	VI	VIII	XI
	S_3	VIII	VII	V	X	VI	VI	VI	XI
	S_4	IV	V	V	VI	V	VII	IX	X
8-9	S_1	V	VII	VII	IX	VII	VII	VIII	IX
	S_2	V	VII	VII	X	VII	V	VIII	IX
	S_3	VIII	VII	VIII	X	VI	VI	IX	XI

M - Measurement division
P - Partitive division

Table E-7

Distribution of Solutions for Cargo Groups Tasks:
Longitudinal Sampling
(N = 20)

Age	Subject	Measurement	Partitive
4	S_1		
	S_2		
5	S_1		
	S_2		*
	S_3		*
6	S_1	*	*
	S_2		*
	S_3	*	*
	S_4	*	*
7	S_1	*	*
	S_2		
	S_3	*	*
	S_4	*	
8	S_1	*	*
	S_2	*	*
	S_3	*	*
	S_4	*	*
9	S_1	*	*
	S_2	*	*
	S_3	*	*

* - Solution

Table E-8

Distribution of Solutions for Animal Groups Tasks: Longitudinal Sampling
(N = 44)

Age	Subject	Measurement	Partitive
	S_1		
	S_2	*	
4	S_3	*	
	S_4		*
	S_5		
	S_1		*
	S_2	*	
5	S_3	*	
	S_4	*	*
	S_5	*	
	S_1	*	*
	S_2	*	
	S_3	*	*
	S_4	*	*
6	S_5	*	*
	S_6	*	*
	S_7	*	*
	S_8	*	*
	S_9	*	*

Age	Subject	Measurement	Partitive
	S_1		
	S_2	*	
	S_3	*	*
	S_4	*	*
7	S_5	*	*
	S_6	*	
	S_7	*	*
	S_8	*	*
	S_9	*	*
	S_{10}	*	
	S_1	*	*
	S_2	*	*
	S_3	*	*
	S_4	*	*
8	S_5	*	*
	S_6	*	*
	S_7	*	*
	S_8	*	*
	S_9	*	*
	S_1	*	*
	S_2	*	*
9	S_3	*	*
	S_4	*	*
	S_5	*	*
	S_6	*	*

* - Solution

Table E-9

Distribution of Solutions for Subjects Attempting
Cargo and Animal Groups Problems in Both Samplings
(N = 20)

		Cross-sectional				Longitudinal			
Age	Subject	C.M.	C.P.	A.M.	A.P.	A.M.	A.P.	C.M.	C.P.
3-4	S_1								
	S_2						*		
4-5	S_1						*		
	S_2			*	*	*	*		*
	S_3			*	*	*			*
5-6	S_1					*	*	*	*
	S_2			*	*	*	*		*
	S_3		*	*	*	*	*	*	*
	S_4					*	*	*	*
6-7	S_1				*	*		*	*
	S_2		*			*			
	S_3	*	*	*	*	*	*	*	*
	S_4	*				*	*	*	
7-8	S_1	*	*	*	*	*	*	*	*
	S_2	*	*	*	*	*	*	*	*
	S_3	*	*	*	*	*	*	*	*
	S_4		*	*	*	*	*	*	*
8-9	S_1	*	*	*	*	*	*	*	*
	S_2	*	*	*	*	*	*	*	*
	S_3	*	*	*	*	*	*	*	*
Total		8	10	12	13	17	16	13	15

* - Solution
C.M. - Cargo measurement
C.P. - Cargo partitive
A.M. - Animal measurement
A.P. - Animal partitive

Table E-10

Distribution of Classification Behaviors Exhibited by Subjects in Animal Groups Problem: Longitudinal Sampling

Age	Subject	M	S	P	S
4	S_1	*	*		
5	S_1	*		*	*
	S_2	*	*		
	S_3			*	*
	S_4	*	*	*	
6	S_1	*	*	*	
	S_2	*	*		
	S_3			*	*
	S_4	*	*	*	*
7	S_1	*		*	
	S_2	*	*	*	*
	S_3	*	*		
8	S_1	*	*		

M - Measurement division
P - Partitive division
S - Solution